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ABSTRACT

A multiplicative (loglinear) model for square tables or other cross-classifications is helpful in locating cells where counts are relatively dense or sparse. This model helps to eliminate the confounding prevalence and interaction effects. It yields a parsimonious set of parameters which describe the table, and goodness of fit can be assessed with standard inferential procedures. A multiplicative specification which fits a particular cross-classification may be obtained in several ways. The model is illustrated using data on the occupational mobility of American men, on the educational attainment of Wisconsin sibling pairs, and on the occupations of male Detroit friendship-pairs. (Author/MK)

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The University of Wisconsin - Madison

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PREFACE

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ABSTRACT

The Structure of Social Relationships: Cross-Classifications of Mobility, Kinship, and Friendship

The paper describes a multiplicative (loglinear) model for square tables (or other cross-classifications) which is helpful in locating cells where counts are relatively dense or sparse. This specification eliminates the confounding of prevalence and interaction effects, which has plagued other schemes for interpreting such tables. The model yields a parsimonious set of parameters which describe the table, and goodness of fit can be assessed with standard inferential procedures. A multiplicative specification which fits a particular cross-classification may be obtained in any of several ways. For example, one may begin with a complete or partial theory about the cross-classification, or one may begin without a theory. By examining residuals from fit under a given model, it is possible to improve the specification in successive rounds of estimation. The counts may be smoothed or aggregated to minimize the chances of fitting and interpreting trivial or unreliable fluctuations in them. Maximum-likelihood estimation is emphasized, but diagnostic information may be obtained using computationally simpler algorithms. The model (and associated inferential methods) can also be used in the comparison of two or more classifications. The exposition is illustrated using data on the occupational mobility of American men, on the educational attainments of Wisconsin sibling-pairs, and on the occupations of male Detroit friendship-pairs.

The general problem may be stated as follows: Having given the number of instances respectively in which things are both thus and so, in they are thus but not so, in which they are so but not thus, and in which they are neither thus nor so, it is required to eliminate the general quantitative relativity inhering in the mere thingness of the things, and to determine the special quantitative relativity subsisting between the thusness and the soness of the things.

Doolittle (1888) as quoted by
Goodman and Kruskal (1959:131)

Social scientists often analyze square tables of counts, where persons, relationships, or other subjects of interest have been classified twice using the same set of categories. For example, in studies of social mobility persons are often classified by their own occupation (or education or social class) and by the occupation (or education or social class) of their fathers. Marriages may be classified by the occupation, education, ethnicity, or religion of each mate. Persons may be cross-classified by place of birth and place of residence, by political party preference before and after an electoral campaign, and so on.

While these square tables of counts may be interpreted and analyzed in many ways, one plausible and traditional interpretation says that each observed count has two components. First, there are effects of the prevalence of observations within each category of the classifications taken singly. For example, in a classification of American men by their own and by their fathers' jobs, one expects to find many blue-collar sons with blue-collar fathers, simply because many men work at blue-collar jobs. Second, there are greater or lesser tendencies for categories to interact, that is, to occur jointly. To continue the example, one expects to find many men in the same occupations as their fathers and few in vastly dissimilar occupations.

The verbal distinction between prevalence and interaction is easy to maintain, but for many years a sound statistical representation of it eluded the efforts of social scientists. The history of this pursuit and the common faults of proposed solutions have been reviewed by Hauser (1978; also, see Featherman and Hauser 1978: Ch. 4). In the next section of this paper I describe a class of multiplicative (loglinear) models whose parameters correspond exactly to the intuitive concepts of prevalence and interaction effects.[1]

Empirically, the correspondence between parameters and concepts only becomes useful when a model of the desired form fits the data. Since one always assumes the existence of prevalence effects for categories of the classificatory variables, the empirical problem is to specify the form of the interactions. Sometimes, sociological theory will provide sufficient guidance in model specification (Goldthorpe and Payne 1978, Hope 1980), but often theory will provide incorrect, incomplete, or contradictory directions. For these reasons I describe methods for assessing goodness of fit and for improving specification through the examination of residuals. After illustrating these ideas in an exploratory analysis of an American father-son occupational mobility table, I show how several conventional statistical analyses of the same table lead to misleading conclusions. Users of empirically based search strategies run the risk of overfitting data; that is, one loses parsimony and reliability by seeking to fit every feature of a sample of observations. In order to minimize such misuses of empirically guided search methods, I have elsewhere described methods of aggregating and smoothing data prior to model selection (Hauser 1979). In the next section of the paper I give two more empirical illustrations of the model: an analysis of similarity in the educational attainments of Wisconsin sibling pairs and an analysis of similarity in the occupational positions held by Detroit men and their friends. The traditional distinction between prevalence and interaction effects is motivated in part by an interest in comparing these components across time and place or between segments of a society. In the last section of the paper I use American mobility data for several cohorts to show how models of the present form may be used to measure and to interpret differences among populations.

A Multiplicative Model of the Mobility Table

My model is a special case of Goodman's (1972c) general multiplicative model for cross-classified data, but I take a slightly different approach from him in developing models of the mobility table. First, I limit my attention to the class of models in which there is only one interaction parameter for each cell in the classification. Second, I do not assume that the categories are ordered. Third, I emphasize the use of exploratory methods in model specification. Elsewhere, these models and methods have been applied in analyses of the 1949 British mobility table (Hauser 1978), of several American mobility tables (Featherman and Hauser 1978: Ch. 4), of Rogoff's (1953) Indianapolis mobility tables (Baron 1977, 1980), of a 1972 British mobility table (Goldthorpe and Payne 1980), and of an American ethnic intermarriage table (Shavit 1978).

Let x_{ij} be the observed frequency in the ij^{th} cell of a classification where $i = 1,\ldots,I$ and $j = 1,\ldots,J$. In the present context the same categories will appear in the same order in rows and columns, and the table will be square with $I = J$. For $k = 1,\ldots,K$, let H_k be a mutually exclusive and exhaustive partition of the pairs (i,j) in which

$$E[x_{ij}] = m_{ij} = \alpha\beta_i\gamma_j\delta_{ij} \qquad (1)$$

where $\delta_{ij} = \delta_k$ for $(i,j)\in H_k$, subject to the normalization $\prod_i \beta_i = \prod_j \gamma_j = \prod_{ij} \delta_{ij} = 1$. The normalization of parameters is a matter of convenience, and we choose the value of α so it will hold. Note that in contrast to the conventional structural model for counted data (Bishop, Fienberg and Holland 1975: Ch. 2), the interaction effects in equation 1 are not constrained within rows or columns even though the marginal frequencies are fitted exactly.[2]

The model says the expected frequencies are a product of an overall effect (α), a row effect (β_i), a column effect (γ_j), and an interaction effect (δ_{ij}). The row and column parameters correspond to the concept of prevalence. For example, in an occupational mobility table they reflect occupational supply and demand, demographic replacement processes, and past and present technologies and economic conditions. The cells (i, j) are assigned to K mutually exclusive and exhaustive subsets, and each of those sets shares a common interaction parameter, δ_k. Thus, aside from total, row, and column effects, each expected frequency is determined by only one interaction parameter, which reflects the density of observations in that cell relative to that in other cells in the table. That is, the interaction parameters of the model correspond directly to the concept of the joint density of observations (White 1963:26), and they may be interpreted as indexes of the social distance between categories of the row and column classifications (compare Rogoff 1953:31-32).

While my model is a special case of Goodman's (1972c, sec. 3) general multiplicative model, unlike several of the models which Goodman (Table V) applied to the British (and Danish) mobility tables, my model does not assume ordinal measurement of occupations.[3] Of course, the assumption of ordinality may help in interpreting results, or empirical findings may be used to explore the metric properties of a classification. The hierarchical dimension is strong in most occupational classifications, and the present applications are artificial in ignoring that.

For the model to be informative, the distribution of levels across the cells of the table must form a meaningful pattern, and one in which the parameters are identified (Mason, Mason, Winsborough and Poole 1973; Haberman 1974:217). Further, the number of levels (K) should be substantially less

than the number of cells in the table. These latter properties are partly matters of substantive and statistical interpretation and judgment, rather than characteristics of the general model or of the data. I have found it difficult to interpret models where the number of levels is much greater than the number of categories recognized in the occupational classification.

It may be helpful to present the model of equation 1 in more than one way. There is a pronounced rightward skew in multiplicative effects because decremental effects are bounded between 0 and 1, while incremental effects are unbounded. It is, for this reason, useful to take logs of frequencies and parameters and to write the model in additive form; then incremental and decremental effects may each range from zero to infinity in absolute value. Let $u^* = \log \alpha$, $u^*_{1(i)} = \log \beta_i$, $u^*_{2(j)} = \log \gamma_j$, $u^*_{12(ij)} = \log \delta_{ij}$, and $u^*_{3(k)} = \log \delta_k$. The model is

$$\log m_{ij} = u^* + u^*_{1(i)} + u^*_{2(j)} + u^*_{12(ij)} \qquad (2)$$

where $u^*_{12(ij)} = u^*_{3(k)}$ for $(i, j) \in H_k$, and H_k is defined as before.[4] Here, the normalization of parameters is $\sum_i u^*_{1(i)} = \sum_j u^*_{2(j)} = \sum_i \sum_j u^*_{12(ij)} = 0$.

A slight variation of equation 1, which I present in multiplicative form, is more suggestive of the way in which I have manipulated empirical data for purposes of estimation and testing. With H_k defined as before, let

$$E[x_{ij}] = m_{ijk} = \alpha\beta_i\gamma_j\delta_k \text{ for } (i, j) \in H_k \qquad (3)$$

and

$$m_{ijk} = 0 \quad \text{for } (i, j) \notin H_k, \qquad (4)$$

subject to the normalization $\prod_i \beta_i = \prod_j \gamma_j = \prod_k \delta_k^{n_k} = 1$, where n_k is the number of

ells assigned to the k^{th} level. This version of the model suggests a -dimensional representation of the original 2-dimensional table in which J(K-1) of the interior cells contain structural zeros, and the original IJ requencies are fitted by row (β_i), column (γ_j) and level (δ_k) parameters, s under a model of quasi-independence (Goodman 1972c:689; Bishop, Fienberg nd Holland 1975:225-226).

To estimate and test models of the present form I have represented cross-lassifications as incomplete multiway arrays, and I have used Fay and oodman's (1973) computer program, ECTA, to estimate frequencies by iterative escaling and to run tests of goodness of fit (and other hypotheses). Under he usual sampling assumptions, e.g., that the data were obtained by independent Poisson or simple multinomial sampling, maximum likelihood estimates are btained in this way (Goodman 1972c:663-667; Bishop, Fienberg and Holland 1975: 06-208). The likelihood ratio test statistic (G^2) computed by the program s asymptotically distributed as X^2 with degrees of freedom equal to IJ, the umber of cells in the array which are not structural zeros, less the number f independent parameters which have been estimated. Often this will be IJ - 1 (I-1) - (J-1) - (K-1) = (I-1) (J-1) - (K-1), but it may be greater, depending on the arrangement of levels within the original 2-way array (Bishop, ienberg and Holland 1975: Ch. 5, esp. 227; Béland and Fortier 1978). Great are should be used in computing degrees of freedom when the design specifies eparable subtables (Bishop, Fienberg and Holland 1975: Ch. 5).

ECTA does not estimate parameters for models of incomplete tables. I ave estimated the (additive) parameters by regressing logs of estimated requencies on dummy-variable representations of the rows, columns, and levels f the design. That is, I created a dummy variable for each row (but one), or each column (but one), and for each level (but one); then I regressed

logs of estimated frequencies on these three sets of variables. By construction this regression completely accounted for the estimated frequencies. I used an auxiliary program to renormalize the parameter estimates as deviations from the grand mean and to compute and display residuals. Using other packaged programs for the analysis of categorical data, one can estimate the models and obtain parameter estimates and measures of fit in a single pass by the methods of maximum likelihood or weighted least-squares (Evers and Namboodiri 1979; Goldthorpe and Payne 1980).

In presenting goodness of fit tests and comparing alternative models, it is convenient to use a single letter to denote each variable. For example, in the next section I let P = father's occupation, S = son's occupation, and H = the levels of interaction to which the several cells in the mobility table are assigned in the model. Following the conventional notation, in which the highest order marginal distributions fitted under a given model are listed in a series of parentheses, I denote the model by (P)(S)(H). Written in this form it is clear that the model is one of statistical independence, conditional on the assignment of cells in the P by S table to levels of H. Under the model the association between P and S is spurious; no association (quasi-independence) between P and S occurs within levels of H (Goodman 1972c:689). One could think of the scheme as a latent factor or latent structure model in which the levels of H are latent classes (Goodman 1974:1231). However, the assignment of cells and hence, of observations to levels of H, is strictly deterministic, so the term "manifest class" might be more fitting.

Mobility to First Jobs of American Men

Table 1 gives frequencies in a classification of son's first, full-time civilian occupation by father's (or other family head's) occupation at son's

16th birthday among American men who were aged 20 to 64 in 1973 and were not currently enrolled in school. The data were obtained in the Occupational Changes in a Generation (OCG) supplement to the March 1973 Current Population Survey (Featherman and Hauser 1975, 1978).[5] Table 2 is a convenient

TABLE 1 ABOUT HERE

display of the final model for the data of Table 1. Each numeric entry in the body of the table gives the level of H_k to which the corresponding entry in the frequency table was assigned; one may think of them as subscripts of dummy variables pertaining to the density of interaction in the several regions of the table. Formally, the entries are merely labels, but for convenience in interpretation the numeric values are inverse to the estimated density of mobility or immobility in the cells to which they refer. I offer no a priori rationale for the specification of interaction effects in Table 2; it is the outcome of a search procedure that I describe later.

TABLE 2 ABOUT HERE

On this understanding the model says that, aside from conditions of supply and demand, immobility is highest in farm occupations (level 1) and next highest in the upper nonmanual category (level 2). If one takes the occupation groups as ranked from high to low in the order listed, one may say there are zones of high and almost uniform density bordering the peaks at either end of the status distribution. There is one zone of high density that includes upward or downward movements between the two nonmanual groups and immobility in the lower nonmanual group. Mobility from lower to upper nonmanual occupations (level 3) is more likely than the opposite movement, and the latter is as likely as stability in the lower nonmanual category

(level 4). Moreover, the densities of immobility in the lower nonmanual category and of downward mobility to it are identical to those in the second zone of relatively high density, which occurs near the lower end of the occupational hierarchy. The second zone includes movements from the farm to the lower manual group and back as well as immobility in the lower manual group. Last, there is a broad zone of relatively low density (level 5) that includes immobility in the upper manual category, upward and downward mobility within the manual stratum, mobility between upper manual and farm groups and all movements between nonmanual and either manual or farm groups. The design says that an upper manual worker's son is equally likely to be immobile or to move to the bottom or top of the occupational hierarchy; obversely, it says that an upper manual worker is equally likely to have been recruited from any location in the occupational hierarchy, including his own.

It is worth noting that four of the five interaction levels recognized in the model occur along the main diagonal, and two of these (levels 4 and 5) are assigned both to diagonal and to off-diagonal cells. Thus, immobility varies among occupational strata, and it is in some cases less likely than mobility. Also, with a single exception the design is symmetric. That is, net of row and column effects upward mobility is more prevalent than downward mobility within the nonmanual group. This asymmetry in the design is striking because it suggests the power of upper white collar families to block at least one type of status loss and because it is the only asymmetry in the design. For example, Blau and Duncan (1967:58-67) suggest that there are semi-permeable class boundaries separating white collar, blue collar, and farm occupations, which permit upward mobility but inhibit downward mobility. The only asymmetry in the present design occurs within one of the broad classes delineated by Blau and Duncan.[6]

Table 3 gives the row, column and interaction effects estimated in the 1973 OCG data under the model of Table 2 for intergenerational mobility to son's first job. The estimates are expressed in additive form; that is, they are effects on logs of frequencies under the model of equation 2. The row and column effects clearly show an intergenerational shift out of farming and into white collar or lower blue collar occupations. These reflect temporal shifts in the distribution of the labor force across occupations, differential fertility, and life-cycle differences in occupational positions. The interaction effects show very large differences in mobility and immobility across the several cells of the classification, and these differences closely follow my interpretation of the display in Table 2. Differences between interaction effects may readily be interpreted as differences in the logs of the estimated frequencies, net of row and column effects. For example, the estimates say that the immobility in farm occupations (at level 1) is 3.40 = 3.044 - (-.356) greater (in the metric of logged frequencies) than the estimated mobility or immobility in cells assigned to interaction level 5 in Table 2. In multiplicative terms, immobility in farm occupations is $e^{3.40}$ = 29.96 times greater than mobility or immobility at level 5. It would be incorrect to attach too much importance to the signs of the interaction effects reported in Table 3, for they simply reflect our normalization rule that interaction effects sum to zero (in the log-frequency metric) across the cells of the table. For example, while the effects of levels 4 and 5 each reflect relatively low densities, it is not clear that either effect indicates "status disinheritance" in the diagonal cells to which it pertains (compare Goodman 1969a, 1969b).

TABLE 3 ABOUT HERE

In any event the effects do show a sharp density gradient across interaction levels. The smallest difference, between levels 3 and 4, indicates a relative

density $e^{.549 - .243} = e^{.306} = 1.36$ times as great at level 3 than at level 4. Immobility in farm occupations and in upper nonmanual occupations is quite distinct from densities at other levels, but also immobility in the farm occupations is $e^{3.044 - 1.234} = e^{1.810} = 6.11$ times as great as in the upper nonmanual occupations.

Overall, the design resembles a valley in which two broad plains are joined by a narrow strip of land between great peaks. The contours of the peaks differ in that the one forming one side of the valley is both taller and more nearly symmetric than that forming the other side. Figure 1 is a pictorial representation of the model whose dimensions are based on my estimates of the row, column, and interaction effects (in multiplicative form). The base of the figure is a unit square; that is, I have renormalized the (multiplicative) row and column effects so the sum of each set is one. Further, the total volume under the surface is one. Thus, length and breadth can be read as probabilities, and height is proportionate to probability. Variations in interaction effects (the vertical dimension) are far larger than those in the horizontal dimension. For this reason the vertical scale has been compressed by a factor of 10, so vertical and horizontal distances are not directly comparable.

FIGURE 1 ABOUT HERE

Evaluating the Model

The model of Table 2 provides less than a complete description of the mobility data in Table 1. Under the model of statistical independence the likelihood-ratio statistic is $G^2 = 6167.7$, which is asymptotically distributed as X^2 with 16 degrees of freedom. With the model of Table 2 as null hypothesis $G^2 = 66.5$ with 12 degrees of freedom, since 4 degrees of freedom are used to

create the five categories of H. By the usual inferential standards the model does not fit, for the probability associated with the test statistic is very small. On the other hand the model does account for 98.9 percent of the association in the data, that is, of the value of G^2 under independence. Given the extraordinarily large sample size, small departures from frequencies predicted by the model are likely to be statistically significant.

Exact tests of the difference between any two interaction parameters can be carried out in a straightforward way. Modify the model to combine the two groups to be contrasted in a single interaction level, and fit the revised model. Since the revised model is a special case of (nested within) the initial model, the difference between the likelihood-ratio x^2 statistics (G^2) of the two models will be distributed as x^2 with one degree of freedom. For example, if I combine levels 1 and 2 of the present model, the revised model yields $G^2 = 676.3$ with 13 df, so I reject the hypothesis that immobility is the same in the farm and upper white collar categories with $G^2 = 676.3 - 66.5 = 609.8$ with 1 df.

By examining errors one can more fully evaluate the fit and perhaps see how to improve the model. Table 4 displays a measure of lack of fit for each cell of the mobility classification. It expresses errors as natural logs of the ratios of observed frequencies to those estimated under the model:

$$\log(e_{ij}) = \log(x_{ij}/\hat{m}_{ij}) = \log x_{ij} - \log \hat{m}_{ij}, \qquad (5)$$

where x_{ij} is the observed frequency and $\hat{m}_{ij}$ is the estimated frequency in the ij^{th} cell. As long as the errors are small, say, less than $\pm.20$, they can be interpreted approximately as proportions. Thus, expressed in this way the errors have a convenient interpretation, and positive and negative deviations are expressed symmetrically in the metric of the (loglinear) model. For

example, the entry of .06 in the cell (3,1) says the observed mobility from upper manual to upper nonmanual occupations is $e^{.06} = 1.06$ times the mobility estimated under the model.[7] The entry of -.17 in the cell (2,3) says mobility from lower nonmanual to upper manual occupations is $e^{-.17} = .84$ times the mobility estimated under the model, i.e., 16 percent less. As suggested by these two examples, the approximation is better when the error is small.

TABLE 4 ABOUT HERE

Under the model of Table 2 cells (1,1), (2,1), and (5,5) are fitted exactly, each by its own parameter. The fourth level--cells (1,2), (2,2), (4,4), (4,5), and (5,4)--is also fitted closely. The largest deviation is the 4 percent underestimate of movement from lower manual to farm occupations. Each other deviation at level 4 is less than one percent. The lack of fit in the model occurs primarily at level 5 of the design. There is a positive deviation of .10 in the one diagonal cell (3,3) assigned to level 5, so immobility in the upper manual (skilled) occupations is not quite so low as in some other cells at the same level. At the same time the largest positive error at level 5 is that for upward mobility from lower manual to lower nonmanual occupations. The two largest negative errors at level 5 pertain to the exchange between upper manual and lower nonmanual occupations (cells (3,2) and (2,3)). Even relative to the low density (presumed by the model) throughout level 5, there is a blockage to movement between the skilled and lower white-collar occupations. This is more striking because there is no similar hindrance to exchange between the skilled and upper white collar occupations (cells (1,3) and (3,1)) or between the lower manual and lower nonmanual occupations (cells (4,2) and (2,4)). From the entries in Table 4 one might argue that the model and the fit could be improved by creating a sixth interaction to include cells (3,2) and (2,3) and, possibly, (1,5), which indicates

a very low rate of mobility from upper nonmanual origins to first jobs in farming.

The indexes of error in Table 4 are in a metric that facilitates interpretation and comparison, but they take no account of sampling variability, which is inverse to expected frequency. Perhaps the simplest way to take account of sampling variability in the errors is to form the ratio

$$z_{ij} = \frac{x_{ij} - \hat{m}_{ij}}{\sqrt{\hat{m}_{ij}}}, \qquad (6)$$

which is the square root of the component of the Pearson chi-square statistic for each cell of the table. The z_{ij} are (roughly) interpretable as unit normal deviates.[8] Since there are several more cells in the table (25) than degrees of freedom under the model (12), the expected value of z_{ij}^2 is considerably less than unity. However, I have not made a correction for that here (see Bishop, Fienberg and Holland 1974:135-141).

Table 5 displays standardized errors from the model of Table 2. Again, one is impressed with the close fit at level 4 and the heterogeneity at level 5 of the model. The interpretation of these errors must be tempered by the results in Table 4, for the standardized errors are not in the metric of the model. Taken in conjunction with earlier results, Table 5 also suggests a respecification of the model in which as a first step cells (2,3), (3,2) and possibly other negative outliers would be assigned to a separate level. However, because the sample is so large, I have not carried the analysis of Table 1 beyond the model of Table 2.

TABLE 5 ABOUT HERE

Mobility Ratios

One other index is particularly useful in evaluating the specification of interaction effects. From equation 1, observed frequencies may be expressed in terms of estimated parameters and errors:

$$x_{ij} = \hat{\alpha}\hat{\beta}_i\hat{\gamma}_j\hat{\delta}_{ij}e_{ij} . \tag{7}$$

Divide both sides of equation 7 by the first three terms on the right-hand side to obtain

$$R^*_{ij} = \frac{x_{ij}}{\hat{\alpha}\hat{\beta}_i\hat{\gamma}_j} = \hat{\delta}_{ij}e_{ij} . \tag{8}$$

I call R^*_{ij} the new mobility ratio, or, simply, the mobility ratio. In the case of diagonal cells R^*_{ij} is equivalent to the new immobility ratio proposed by Goodman (1969a, 1969b, 1972c; also, see Pullum 1975:7-8), but I suggest the ratio be computed for all cells of the table as an aid in the evaluation of model design. If the model is specified correctly, the estimated interactions ($\hat{\delta}_{ij}$) will be more useful in interpretation than the R^*_{ij}, for the latter will confound interaction effects with sampling errors (e_{ij}). On the other hand, when the model is not correctly specified, the errors (e_{ij}) will reflect specification error as well as sampling variability. For this reason the R^*_{ij} can be useful in revising a model which does not fit the data.

To illustrate the use of the R^*_{ij}, Table 6 gives these indexes for the counts of mobility to first jobs. Obviously, the pattern of the R^*_{ij} conforms to our earlier description of the design. Moreover, as may not have been obvious from the $\hat{\delta}_{ij}$ (Table 3) and the e_{ij} (Table 4) taken separately, the fit is good enough so there is no overlap in interactions across levels recognized in the design. For example, if immobility among skilled workers--cell (3,3)--is high relative to mobility in other cells at level 5 in

TABLE 6 ABOUT HERE

Table 2, the immobility in that category is still substantially less than the immobility in any other occupation group. Again, level 5 appears to be heterogeneous, but I have not carried the analysis of Table 1 beyond the model of Table 2.

Conceptually, R^*_{ij} is related to R_{ij}, Rogoff's (1953) social distance mobility ratio and Glass's (1954) index of association:

$$R_{ij} = \frac{x_{ij}}{(1/N)\, x_{i.}\, x_{.j}} , \tag{9}$$

where N is the sum of observed counts, and $x_{i.}$ and $x_{.j}$ are, respectively, sums of counts in the i^{th} row and in the j^{th} column. Both R_{ij} and R^*_{ij} may be interpreted as ratios of observed counts to those estimated from a scale factor and row and column effects under a given statistical model (see Hauser 1978:92?-924). Indeed, $R_{ij} = R^*_{ij}$ in the special case of the model of simple statistical independence, which specifies no interaction effects. In terms of equation 1, R^*_{ij} becomes R_{ij} when we specify $\delta_{ij} = 1$ for all i and j.

As a measure of interaction, R_{ij} has several undesirable properties. Although R_{ij} was supposed to be independent of prevalence (row and column) effects (Rogoff 1953:32), it varies inversely as the marginal proportions in the i^{th} row and j^{th} column. The maximum of R_{ij} is the reciprocal of the larger of the marginal proportions in the i^{th} row and j^{th} column. Also, the set of R_{ij} for a square table determines the row and column marginal distributions. This renders R_{ij} useless in comparing interaction effects across tables with differing marginal distributions, for the multiple sets of R_{ij} cannot take on values corresponding to the hypothesis of no change. Further, the R_{ij} cannot be symmetric across the main diagonal ($R_{ij} = R_{ji}$)--showing,

for example, equal propensities toward upward and downward mobility -- unless the observed counts are symmetric ($x_{ij} = x_{ji}$). Thus, propensities toward upward and downward mobility cannot appear to be the same unless the frequencies of upward and downward mobility are the same, and, consequently, the two marginal distributions are the same (Blau and Duncan 1967:93-97, Tyree 1973). These undesirable properties all arise because, when the model of simple statistical independence does not fit the data, R_{ij} confounds prevalence effects (of rows and columns) with interaction effects (Goodman 1969b). That is, the important difference between R_{ij} and R^*_{ij} is that the new mobility ratio is obtained from a model that fits the data, so row and column effects are not confounded with relative densities (interactions) in the interior of the table. For these reasons R^*_{ij} does not have the undesirable properties of R_{ij}. In general, (1) R^*_{ij} is not bounded; (2) in a square table the set of R^*_{ij} do not determine the marginal frequencies (nor the marginal effects); and (3) the set of R^*_{ij} can be symmetric, i.e., $R^*_{ij} = R^*_{ji}$, under any set of marginal frequencies (or effects) (compare Tyree 1973: 577-580).

In this context it is instructive to show the relationship between the parameters of the multiplicative model and the marginal frequencies of the mobility table. The model fits the observed marginal frequency distributions, that is, $\sum_i \hat{m}_{ij} = x_{.j}$ and $\sum_j \hat{m}_{ij} = x_{i.}$, so

$$\sum_i \hat{m}_{ij} = \sum_i \hat{\alpha}\hat{\beta}_i\hat{\gamma}_j\hat{\delta}_{ij} = \hat{\alpha}\hat{\gamma}_j\sum_i\hat{\beta}_i\hat{\delta}_{ij} = x_{.j} \tag{10}$$

and

$$\sum_j \hat{m}_{ij} = \sum_j \hat{\alpha}\hat{\beta}_i\hat{\gamma}_j\hat{\delta}_{ij} = \hat{\alpha}\hat{\beta}_i\sum_j\hat{\gamma}_j\hat{\delta}_{ij} = x_{i.} \quad . \tag{11}$$

Thus, the marginal frequency in a given column (or row) is the product of the corresponding column (or row) effect, a scale factor, and a weighted sum of the row (or column) effects, where the weights are the interaction effects for corresponding rows (or columns) within the given column (or row).

Similarly, from equations 7 and 8 we can write

$$\sum_i x_{ij} = \hat{\alpha}\hat{\gamma}_j\sum_i\hat{\beta}_i\hat{\delta}_{ij}e_{ij} = \hat{\alpha}\hat{\gamma}_j\sum_i\hat{\beta}_i R^*_{ij} = x_{.j} \tag{12}$$

and

$$\sum_j x_{ij} = \hat{\alpha}\hat{\beta}_i\sum_j\hat{\gamma}_j\hat{\delta}_{ij}e_{ij} = \hat{\alpha}\hat{\beta}_i\sum_j\hat{\gamma}_j R^*_{ij} = x_{i.} \quad , \tag{13}$$

so one may alternatively think of the new mobility ratios as weights in the expressions relating marginal frequencies to corresponding marginal effects. There are expressions in the old mobility ratios, R_{ij}, which are formally similar to equations 12 and 13; however, those expressions can be simplified to eliminate the R_{ij}, while equations 12 and 13 cannot be simplified to eliminate the R^*_{ij}. For example, from the definition of R_{ij},

$$x_{ij} = \frac{1}{N}x_{i.}x_{.j}R_{ij} \quad , \tag{14}$$

so

$$\sum_i x_{ij} = \frac{x_{.j}}{N}\sum_i x_{i.}R_{ij} = x_{.j} \quad , \tag{15}$$

for by definition $\sum_i x_{i.}R_{ij} = N$.

Suppose it were possible to solve for the marginal effects by writing linear equations in the R^*_{ij}, so (following Blau and Duncan 1967:93-94):

$$\sum_i \hat{\beta}_i R^*_{ij} = m \text{ for all } j \tag{16}$$

and

$$\sum_j \hat{\gamma}_j R^*_{ij} = n \text{ for all } i. \tag{17}$$

Under these conditions equations 12 and 13, respectively, can be rewritten as

$$x_{.j} = \hat{\alpha}\hat{\gamma}_j m \tag{18}$$

and

$$x_{i.} = \hat{\alpha}\hat{\beta}_i n \quad . \tag{19}$$

That is, if the mobility ratios determine the marginal effects, then $x_{.j}$ is just a scalar multiple of γ_j, and $x_{i.}$ is just a scalar multiple of β_i, which is to imply the model is indistinguishable from the simple independence model and R^*_{ij} is indistinguishable from R_{ij}. But in general the model is not that of simple independence; R^*_{ij} is not equal to R_{ij}; and the row and column parameters are not scalar multiples of the marginal frequencies. This says that the row and column effects under the model are not generally determined by the R^*_{ij}. In summary, the new mobility ratios appear to have properties that make them useful in model specification.

Mobility Ratios and Other Measures of Interaction

Do the substantively novel features of my interpretation of mobility to first jobs merely reflect peculiarities of the 1973 OCG data? Obversely, are those features a consequence of a different way of looking at the data? If the old mobility ratios (R_{ij}) provide misleading clues about the structure of mobility tables, are there valid measures which are easier to obtain than the new mobility ratios (R^*_{ij})?

In this section I attempt to answer these questions by subjecting the 1973 OCG data of Table 1 to a number of alternative analyses. In brief, the answers are as follows. The 1973 OCG table of mobility to first jobs is generally similar to other mobility tables, and any novelty in my conclusions arises from the use of my structural model. Moreover, I have directed my criticisms of mobility indexes primarily at the old mobility ratio, R_{ij}, but several other common measures of association also fail to elucidate the

pattern of association in the mobility table for much the same reasons that R_{ij} is defective. Further, by obtaining new mobility ratios (R^*_{ij}) under relatively simple models of quasi-independence (which are special cases of the model of equation 1), I can diagnose the pattern of association without positing a model for the full table. In some cases one can obtain sound diagnostic information without extensive calculation.

Table 7 shows standard outflow and inflow tables based on the data of Table 1. The 1973 OCG table of mobility to first jobs appears to resemble other mobility tables, such as the 1962 and 1973 OCG tables for mobility to current jobs (Hauser and Featherman 1976; U.S. Department of Health, Education, and Welfare 1969). There is evidence of status persistence and self-recruitment; the latter is especially strong in the case of farm occupations. There is also substantial short-distance mobility. Last, except for the prevalence of lower manual first occupations--which is greater than the prevalence of lower manual fathers' occupations--the flow of manpower is primarily from lower to higher levels of the occupational hierarchy, and there is a marked decline in the proportion of men with farm occupations relative to the proportion with farm origins.[9]

TABLE 7 ABOUT HERE

Again, Table 5 may be helpful in evaluating other measures of association when the latter are presented in multiplicative form. One previously unmentioned feature of the array of R^*_{ij} is their high degree of symmetry across the main diagonal (with the marked exception of the interchange between upper and lower nonmanual occupations). This symmetry of upward and downward flows is not apparent in the inflow and outflow tables, nor are many of the other patterns of association in Table 6. One possible exception is the relatively high degree of immobility in the upper nonmanual and farm occupations. In

any event I shall not pursue the comparison of Tables 6 and 7, for neither the inflow nor outflow tables are purported to free the pattern of association from the influence of both marginal distributions.

Old Mobility Ratios

Table 8 gives the old mobility ratios (R_{ij}) for the data of Table 1 under the model of simple statistical independence. Clearly, one need not resort to hypothetical data to show the differences between interpretations based on the old and new mobility ratios. The entries in Table 8 are similar to those which an experienced student of mobility has encountered on many occasions; for example, compare Pullum's (1975:3-7) description of the 5 X 5 British mobility table. From the R_{ij} one would conclude (correctly) that there is substantial immobility at both the top and bottom of the occupation hierarchy, but not nearly as much immobility as is indicated by the R^*_{ij}. The R_{ij} also show status immobility in the three middle occupation groups, but less in the lower manual than in the other two categories. In contrast the R^*_{ij} show a very low level of immobility in the upper manual group, and they show moderate and roughly equal levels of immobility in the lower nonmanual and lower manual groups. Both sets of ratios show greater than expected interchange between the upper and lower nonmanual groups with the upward flow exceeding the downward flow. The R_{ij} show asymmetric flows between the lower manual and farm groups, both of which are below expectations, but between these same two groups the R^*_{ij} show roughly equal flows which are larger than those expected from row, column, and scale effects. With a single exception the R_{ij} decline regularly as one moves away from the main diagonal in any row or column, but the R^*_{ij} are low and fluctuate irregularly outside the eight cells in the upper left and lower right corners of the table. Outside those same [illegible]rs four of the R_{ij} (in cells (3,2), (3,3), (3,4), and (4,2)) show

greater frequencies than expected, but none of the R^*_{ij} show greater frequencies than expected. Last, with a single exception the R_{ij} are greater in size in corresponding cells below than above the main diagonal, and this suggests a preponderance of upward relative to downward mobility. At the same time, the R^*_{ij} are roughly the same size in corresponding cells above and below the main diagonal with the exception of the one asymmetry in the specification of the model.

TABLE 8 ABOUT HERE

The relationship between the R_{ij} and R^*_{ij} can be clarified by expressing the former in terms of the latter. By definition

$$R_{ij} = \frac{x_{ij}N}{x_{i.}x_{.j}} \qquad (20)$$

Under the model of equation 1 and from equations 7 and 8 I write

$$x_{ij} = \hat{\alpha}\,\hat{\beta}_i\,\hat{\gamma}_j\,R^*_{ij}\ , \qquad (21)$$

so

$$N = \sum_i \sum_j x_{ij} = \hat{\alpha} \sum_i \sum_j \hat{\beta}_i\,\hat{\gamma}_j\,R^*_{ij}\ . \qquad (22)$$

By substitution from equations 21, 22, 12, and 13 I rewrite equation 20 as

$$R_{ij} = \frac{(\hat{\alpha}\,\hat{\beta}_i\,\hat{\gamma}_j\,R^*_{ij})(\hat{\alpha}\sum_i\sum_j \hat{\beta}_i\,\hat{\gamma}_j\,R^*_{ij})}{(\hat{\alpha}\,\hat{\beta}_i\sum_j \hat{\gamma}_j\,R^*_{ij})(\hat{\alpha}\,\hat{\gamma}_j\sum_i \hat{\beta}_i\,R^*_{ij})} = \frac{R^*_{ij}\,(\sum_i\sum_j \hat{\beta}_i\,\hat{\gamma}_j\,R^*_{ij})}{(\sum_j \hat{\gamma}_j\,R^*_{ij})(\sum_i \hat{\beta}_i R^*_{ij})} \qquad (23)$$

The double sum in the numerator of equation 23 is a scale factor which does not vary with the indexes i and j. Thus, the variable parts of the expression say that R^*_{ij} is related to R_{ij} inversely as the product of weighted averages of the column and of the row parameters, whose respective weights are the

new mobility ratios in the i^{th} row and the j^{th} column. In general R_{ij} will be low, relative to R^*_{ij}, when the new mobility ratios in the i^{th} row and the j^{th} column are large, and R_{ij} will be high, relative to R^*_{ij}, when the new mobility ratios in the i^{th} row and the j^{th} column are small. For example, the relatively large value of R_{33}, the old immobility ratio for upper non-manual (skilled) occupations, is explained by the very low levels of association throughout the third row and the third column of the table (when that association is indexed by R^*_{ij}). In general a given row and column need not contain only high or only low R^*_{ij}, and the relationship of R_{ij} and R^*_{ij} will vary among cells in the mobility classification.

Standardized Deviates

Are other expressions of residuals under the model of simple independence more instructive with regard to the structure of association in the mobility table? The upper panel of Table 9 shows standardized deviates under the model of statistical independence (recall equation 6 and Table 5). In looking at the standardized deviates we have left the (multiplicative) metric of the model. That is, the standardized deviates are test statistics, and they reflect variations in the standard errors of deviations of observed from expected frequencies across cells of the table, as well as the pattern of the residuals themselves. Thus, we would expect the standardized deviates to be more helpful to us in locating extreme outliers, as in cells (1,1) and (5,5), than in evaluating the pattern of association in the table. In any event the pattern exhibited by the standardized deviates is rather close to that of the old mobility ratios. Similar results would be obtained had we chosen to express the residuals as components of the likelihood-ratio statistic (G^2) or as Freeman-Tukey deviates (Bishop, Fienberg and Holland 1975:136-137; but [...]rntz 1978).

TABLE 9 ABOUT HERE

Haberman (1973) has suggested a transformation of the standardized deviates, which is in our notation

$$d_{ij} = \frac{z_{ij}}{(1 - x_{i.}/N)(1 - x_{.j}/N)} \quad . \qquad (24)$$

The adjusted residual, d_{ij}, has better asymptotic distributional properties than z_{ij}, and Brown (1974) has shown that d_{ij} is more effective than z_{ij} in identifying a small number of outliers. The adjusted standardized deviates are shown in the lower panel of Table 9. As one might expect the adjusted deviates are not more instructive with regard to the overall pattern of association than are the unadjusted deviates or the old mobility ratios. At the same time they do clearly identify three large positive deviates (in cells (1,1), (4,4), and (5,5)) whose elimination might elucidate the pattern of association throughout the table; I shall elaborate this suggestion later.

Parameters of the Saturated Loglinear Model

Like the specification of equation 2, the conventional parametric representation of the loglinear model also describes frequencies in terms of parameters for row effects, column effects, and interaction effects. Moreover, one can "saturate" the model by including all main effects and interactions, thus fitting observed counts perfectly. Critics have suggested to me that interaction parameters under the saturated loglinear model would yield, by inspection alone, substantially the same interpretation as that obtained using my model. However, the usual normalization of parameters of the loglinear model gives priority to row and column effects relative to interactions, that is, relative to association in the interior cells of the table. Even though the saturated loglinear model fits a table completely, this conventional

normalization of parameters has much the same effect on the pattern of interaction parameters as the assumption of statistical independence has on the pattern of old mobility ratios. Consequently, under the saturated loglinear model the multiplicative parameters for the interactions are no more informative than the residuals from the simple independence model: the marginal effects are too large in rows or columns where the interactions are strong, and the marginal effects are too small where the interactions are weak. Obversely, the estimated interactions are deflated or inflated relative to a model in which row, column, and interaction effects are given equal priority.

An algebraic presentation of the conventional loglinear model may clarify this argument. Let

$$l_{ij} = \log m_{ij} , \tag{25}$$

where m_{ij} is the expected count in the ij^{th} cell, and $\sum_i \sum_j m_{ij} = N$. The saturated loglinear model says that

$$l_{ij} = u + u_{1(i)} + u_{2(j)} + u_{12(ij)} , \tag{26}$$

subject to the constraints

$$\sum_i u_{1(i)} = \sum_j u_{2(j)} = \sum_i u_{12(ij)} = \sum_j u_{12(ij)} = 0. \tag{27}$$

Under these constraints the u-terms are obtained by a row and column decomposition of the logs of expected frequencies paralleling that in a two-way analysis of variance with one observation per cell (Bishop, Fienberg and Holland 1975:24):

$$u = \frac{\sum_i \sum_j l_{ij}}{I J} , \tag{28}$$

$$u_{1(i)} = \frac{\sum_j l_{ij}}{J} - u , \tag{29}$$

$$u_{2(j)} = \frac{\sum_i l_{ij}}{I} - u , \tag{30}$$

and

$$u_{12(ij)} = l_{ij} - (u_{1(i)} + u_{2(j)}) + u. \tag{31}$$

Note that equation 26 and equation 2 are identical (excepting notation), and the constraints on row and column parameters are the same in the two models. The important difference between the models of equations 2 and 26 lies in the constraints on the interaction parameters ($u^*_{12(ij)}$ and $u_{12(ij)}$) and in the implications of those constraints for the specification of equalities among subsets of interaction parameters. The specification that the interaction parameters sum to zero within every row and within every column of the table (see equation 27) is equivalent to the model of simple independence in its implications for interpreting the pattern of association in the table. By relaxing the normalization of the $u_{12(ij)}$ in equation 27 one can obtain new insights into the pattern of association in the table.

This observation can be elaborated by writing the main effects in the saturated model (or equations 26 and 27) as functions of the main effects and and interactions in equation 2. In the loglinear metric I substitute equation 2 for l_{ij} in each of equations 28 to 31, recalling the constraints on sums of the u^*-terms:

$$u = u^* \tag{32}$$

$$u_{1(i)} = u^*_{1(i)} + \sum_j u^*_{12(ij)}/J \tag{33}$$

$$u_{2(j)} = u^*_{2(j)} + \sum_i u^*_{12(ij)}/I \qquad (34)$$

and

$$u_{12(ij)} = u^*_{12(ij)} - [\sum_j u^*_{12(ij)}/J + \sum_i u^*_{12(ij)}/I] + 2u^* . \qquad (35)$$

The constants in the two models are the same; the main effects of the i^{th} row differ by the average of the interaction effects in that row; the main effects of the j^{th} column differ by the average of the interaction effects in that column; the interaction effects of the ij^{th} cell differ by a constant less the averages of the interaction effects in the i^{th} row and the j^{th} column. Note that equations 33 and 34 are (additive) analogs of equations 10 and 11, respectively. Just as the marginal sum of frequencies in any row (or column) varies with the interaction effects in that row (or column), so the main effect of any row (or column) in the saturated model varies with the interaction effects in that row (or column). Similarly, equation 35 is an (additive) analog of equation 23. Just as R_{ij} varies (inversely) relative to R^*_{ij} as a function of the interactions in the i^{th} row and j^{th} column, so $u_{12(ij)}$ varies (inversely) relative to the $u^*_{12(ij)}$ as a function of the interactions in the i^{th} row and j^{th} column. If we regard equation 2 as the structural model, then the conventional row by column decomposition of the saturated model yields main effects and interactions which are mixtures of parameters of the structural model (compare Goldberger 1973 or Duncan 1975:151).

For example, Table 10 presents the multiplicative parameters for a saturated model which fits the data of Table 1. These tell essentially the same story as the old mobility ratios in Table 8, but they are a slight improvement over the old mobility ratios. Note that the parameters in cells (1,1) and (5,5) are larger than the old mobility ratios in Table 8, and the ...ons in parameters of the sparser cells is less than that among mobility

ratios in the corresponding cells. The improvement occurs because the row and column effects under statistical independence are based on sums of frequencies, while they are based on sums of logs of frequencies in the case of the saturated loglinear model. The operation of taking logs reduces the effect of positive outliers on the row and column sums, and so reduces (but does not eliminate) the influence of the small number of large interactions, i.e., of the positive skew of frequencies, on the row and column effects. However, the array of parameters in Table 10 is still substantially misleading in respect to the pattern of association in Table 1. For example, it shows roughly equal immobility in cells (2,2), (3,3), and (4,4) and it does not suggest the homogeneity of densities in cells assigned to levels 4 or 5 in Table 2.

TABLE 10 ABOUT HERE

Adjustment to Uniform Marginals

I shall consider one other method of inspecting the pattern of association in the full mobility table. Mosteller (1968) drew attention to Levine's (1967) use of iterative proportional rescaling to adjust British and Danish 5 x 5 mobility tables to uniform marginal distributions. This adjustment facilitated Levine's interpretation of the mobility tables and exposed similarities in the pattern of association in the two tables. (For further evidence and discussion of the similarity of the British and Danish tables see Goodman 1969a, 1969b and Bishop, Fienberg and Holland 1975:100.) The iterative proportional rescaling procedure is generally attributed to Deming, who suggested it be used to adjust sample cross-classifications to known marginal distributions (Scheuren and Oh 1975); the same procedure is used to obtain maximum-likelihood estimates of frequencies in loglinear models of contingency tables when no closed-form estimates exist (Bishop, Fienberg and Holland 1975:83-87). The method has been applied frequently in recent years

(Duncan 1966; Tyree and Treas 1974; Garnier and Hazelrigg 1974, 1976; Hazelrigg 1974a, 1974b; Hauser et al. 1975a; Pullum 1975; Hauser, Featherman and Hogan 1977).

The adjustment procedure is straightforward. Each row entry is multiplied by the ratio of the desired row sum to the actual row sum. Then each column entry is multiplied by the ratio of the desired column sum to the actual column sum. By alternating row and column adjustments, convergence of the adjusted cell counts to both desired marginal totals is usually obtained within a few iterations. Multiplicative adjustment preserves the initial pattern of association in a table because odds-ratios are invariant to scalar transformations applied uniformly across rows and columns. For example, the upper panel of Table 11 gives hypothetical frequencies in a 2 x 2 mobility table. The odds-ratio in this table is

$$\frac{x_{11}/x_{12}}{x_{21}/x_{22}} = \frac{x_{11}/x_{21}}{x_{12}/x_{22}} = \frac{x_{11}\ x_{22}}{x_{21}\ x_{12}} \quad . \qquad (36)$$

Suppose the frequencies in interior rows 1 and 2 of the table are multiplied by arbitrary non-zero constants, say, a and b, respectively. Likewise, the frequencies in the interior columns of the table are multiplied by non-zero constants c and d. The adjusted frequencies are shown in the lower panel of Table 11. Under this transformation the marginal proportions will not generally be preserved, but the odds-ratio will be unaffected, for

$$\frac{(ac\ x_{11})(bd\ x_{22})}{(ad\ x_{12})(bc\ x_{21})} = \frac{x_{11}\ x_{22}}{x_{12}\ x_{21}} \quad . \qquad (37)$$

As early as 1912 Yule recognized the desirability of constructing measures of association with this invariance property (Goodman and Kruskal 1954:747). In fact both the old (R_{ij}) and the new (R^*_{ij}) mobility ratios have this

invariance property, i.e., that they preserve the odds-ratios in the observed frequencies; this is obvious from inspection of equations 20 and 21.

<u>TABLE 11 ABOUT HERE</u>

In recommending adjustment to uniform marginals Mosteller (1968:8) implied that the adjusted frequencies elucidated the pattern of association in a table:

> ...we can interpret the resulting numbers as transitional or conditional probabilities expressed in per cents--either son's distribution given the father's category, or father's given the son's... In the sense of having a common nucleus of association...it would be fair to say that the two occupational tables are nearly equivalent.

Similarly, in respect to the same example, Bishop, Fienberg and Holland (1975:100) write

> By comparing diagonal values we see that, except for category 1, the tendency for fathers and sons to fall into the same category is stronger in Denmark than Britain. By looking across rows we can see, for fathers in each category, in which country the sons are more mobile.

Again, Fienberg (1971:308) suggests that standardization to uniform marginals permits one "to look at the association or interaction, unconfounded by the two sets of marginal allocations."

While it is strictly correct that the original marginal frequencies of a table cannot be deduced from the set of frequencies adjusted to uniform marginals, neither do the adjusted frequencies display the pattern of association in the sense intended here. For example, Table 12 gives the adjusted frequencies of the data in Table 1. I chose uniform marginal sums of 5, so the condition of simple independence would yield an entry of unity in each cell (compare Tyree and Treas 1974). The pattern of marginally adjusted frequencies in Table 12 is virtually identical to that of the old mobility ratios in Table 8, and it is markedly different from the pattern of the new

mobility ratios in Table 6. One improvement is that the adjusted frequencies are more nearly symmetric about the main diagonal than are the old mobility ratios. If the marginal adjustment eliminates variation in the marginal distributions, why doesn't it uncover the underlying pattern of association in the table? The problem lies in the distinction between marginal distributions ($x_{i.}$, $x_{.j}$) and marginal effects (β_i, γ_j); recall equations 10 and 11. Equalization of the marginal distributions does not equalize the marginal effects; the former differ from the latter because they are confounded with the underlying pattern of interaction in the table. For example, consider again the third row or the third column of the mobility classification (upper manual occupations). Because the interactions are weak in that row and column, the marginal proportions are relatively low. Consequently, the adjustment procedure induces too large a relative increase in the marginal frequencies in that row and column, leading to an excessively large adjusted entry in the (3,3) cell.[10]

TABLE 12 ABOUT HERE

Similarly, in analyzing the British and Danish mobility tables Levine (1967) adjusted the frequencies to uniform marginals, took logs of the adjusted frequencies, and fitted smooth curves to the logs of adjusted frequencies. Levine's model is flawed because the initial multiplicative adjustment did not reveal the pattern of association in the British and Danish tables. However, if adjustment of the full table to uniform marginals does not yield the interaction structure, neither is it valueless. The procedure can be used as a rough guide to similarity or dissimilarity in the odds-ratio of the two or more tables, even though it does not provide a satisfactory picture of the pattern of association in any one classification.

In summary, I have evaluated several measures of association which are based on the model of simple independence and other measures which are based on the saturated model. There are real differences among these measures of association. For example, Haberman's adjusted deviates are useful in locating a small number of outlying frequencies. When the interaction structure is symmetric but the marginal distributions are not, the marginally adjusted frequencies or the multiplicative parameters of the saturated model will display the underlying symmetry, but the old mobility ratios will not. At the same time, each of the measures I have reviewed suggests essentially the same interpretation of the pattern of association in the mobility table. This interpretation is in each case fundamentally different from that suggested by the new mobility ratios (R^*_{ij}). This difference occurs primarily because the other measures of association confound main effects (of rows and columns) with interaction effects.

Model Specification Under Quasi-Independence

One can obtain superior insights into the structure of association in a table by temporarily ignoring those cells of the classification which contribute most to the confounding of interaction effects with row and column effects. In Goodman's (1965, 1969a) terms one "blanks out" those cells and fit models of quasi-independence to the remaining cells. Equivalently, in my multiplicative models for the full table, I fit one parameter to each cell which is to be ignored, and I assign the remaining cells to a single level of the design matrix. Table 13 shows the equivalent design matrices for three models of quasi-independence in the 5 x 5 table.

TABLE 13 ABOUT HERE

Model Q1 is the quasi-perfect mobility model. It ignores (or fits exactly) the frequencies on the main diagonal, which represent occupational inheritance relative to the five-category occupational classification. Under the null hypothesis there is no association in the remainder of the table, which is coded at level 1 in the design matrix both for the full and the partial tables. Frequencies are estimated in those cells by iteratively fitting a matrix containing ones at level 1 and zeros elsewhere, i.e the Q1 design matrix in the left-hand column of Table 13, to the observed marginal frequencies in the fitted cells. That is, the fitting procedure preserves both the observed marginal frequencies and the hypothesized lack of association in the fitted portion of the table. While there are 25 degrees of freedom in the 5 x 5 table, we lose nine degrees of freedom in fitting row, column, and scale effects, and we lose another five degrees of freedom in fitting the six-level Q1 model. Thus, under the null hypothesis that there is no association off the main diagonal, there are 25 - 9 - 5 = 11 degrees of freedom for error.

Table 14 summarizes the fit of maximum likelihood estimates of the independence model and other multiplicative models of the 5 x 5 table of mobility to first jobs (Table 1). Clearly Model Q1 accounts for much of the association in the table. While $G^2 = 683.06$ is still very large relative to its degrees of freedom, it is only about one-ninth the value of G^2 under simple independence. Further, while the simple independence model misallocates 20.1 percent of the joint distribution of fathers and sons (as indicated by the index of dissimilarity, Δ, in the fourth column of Table 14), Model Q1 misallocates only 5.5 percent of the observations.

TABLE 14 ABOUT HERE

The first panel of Table 15 presents ratios of observed frequencies to those expected at level 1 of Model Q1, i.e., within the zone of quasi-independence specified in the design matrix. Although the diagonal cells are not assigned to level 1 of Model Q1, we have also shown ratios of observed to expected frequencies in the diagonal cells. The diagonal entries are the indexes of immobility proposed by Goodman (1969a, 1969b); they are ratios of the observed frequencies to those frequencies which would have been estimated in the main diagonal if the quasi-independence hypothesis held in the main diagonal. Alternatively, we may say they are the frequencies predicted by the row, column, and scale effects at level 1 of the design.

Those expected frequencies are not produced directly by the computer program (ECTA) used to estimate Models Q1, Q2, and Q3, but with a simple model in a small table it is convenient to estimate the expected frequencies in those cells from the expected frequencies in the level where quasi-independence is presumed to hold. Under the null hypothesis all of the odds-ratios within the zone of quasi-independence are equal to unity (Goodman 1968, 1969a). Thus, if we know only three expected frequencies in a 2 x 2 subtable of the full table, we can solve for the fourth expected frequency by setting the odds-ratio in the subtable equal to one. For example, Table 16 shows the expected frequencies in each cell of the mobility table under Model Q1. To obtain the expected frequency in cell (1,1), we could use the entries in cells (1,2), (3,1), and (3,2) to write

$$m_{11} = \frac{754.9\ (344.1)}{697.7} = 372.3 \qquad (38)$$

Other combinations of cells could be used to obtain the same estimate within the limits of rounding error. In models (like Q3) where a relatively large number of cells has been ignored, it may take a certain amount of ingenuity

to fill in the expected frequencies for all of the blanked-out cells.[11]

TABLES 15 AND 16 ABOUT HERE

The ratios of observed to expected frequencies in Table 15 are not new mobility ratios (R^*_{ij}), but they differ from the R^*_{ij} only by a scalar multiple. That is, I have expressed the R^*_{ij} as deviations from a scale factor (or grand mean) for the full table, but the ratios in Table 15 are expressed as deviations from expected frequencies at level 1 of the design. The relationship between those ratios and the R^*_{ij} is like that between the normalization of parameters in dummy-variable regression and in multiple classification analysis. In the former case the reference point is the effect of one category of a qualitative regressor, and in the latter case the reference point of effect measures is the grand mean of the dependent variable. With the understanding that a change in normalization has occurred, I shall refer to the entries in Table 15 as mobility ratios.

Under Model Q1 the mobility ratios show a pattern of association which is somewhere between that displayed by the R_{ij} (Table 8) and the R^*_{ij} (Table 6). Relative to the R_{ij}, the ratios in Table 15 are larger in cells (1,1) and (5,5), and they are also relatively larger in cells (4,5) and (5,4) and, to a lower degree, in cells (1,2) and (2,1). The ratios for Model Q1 do not appear to fall as rapidly as one moves away from the main diagonal as do the R_{ij}. At the same time there is still a relatively high ratio in the central diagonal cell, (3,3).

The fit of Model Q1 is not very close, and there are relatively large mobility ratios in four of the cells which were not fitted exactly under Model Q1--(1,2), (2,1), (4,5), and (5,4). For these reasons I write the design matrix of Model Q2 to ignore those four cells as well as the diagonal cells.

Thus, Model Q2 has only seven degrees of freedom for error under the null hypothesis. White (1963) and Pullum (1975) advocate models of this form; also, see Fienberg (1976) for an evaluation of this specification as applied by Pullum. The fit is much improved under Model Q2, so one would expect the residuals to be more informative. Under Model Q2, G^2 = 50.05, which is only 0.8 percent of its value under simple independence. The model misclassifies only 1.4 percent of the joint frequency distribution of fathers' and sons' occupations.

The mobility ratios for Model Q2 show a pattern which is far more like that of the R^*_{ij}. One problem is the relatively high ratio in cell (3,3), but there is otherwise little variation in the ratios outside the intersections of rows 1 and 2 with columns 1 and 2 and of rows 4 and 5 with columns 4 and 5. Moreover, the change in specification has again increased the mobility ratios in cells (1,1) and (5,5).

I make one other effort to fit the data more closely without precluding the estimation of all of the row and column effects, that is, without making it impossible to obtain mobility ratios for all of the cells in the table. In Model Q3 I ignore all of the cells on the main diagonal and on the adjacent minor diagonals. Here the fit is rather close with G^2 = 15.7 with three degrees of freedom and Δ = 0.6. Of course, in obtaining this fit I ignore (or fit exactly) the cells containing about three-fourths of the observations, but my purpose is not to fit the data both closely and parsimoniously. Rather, I am trying to explore association in the table by fitting it closely in a way that permits me to obtain diagnostic measures of association.

Clearly, relative to the standard set by the pattern of R^*_{ij} in Table 6, Model Q3 is very helpful in uncovering the pattern of association in Table 4. The mobility ratios in Panel Q3 of Table 15 show all of the major features of

the display in Table 2 and of the R^*_{ij} in Table 6. Taken in conjunction with my review of other residual measures, the lesson in this illustrative analysis should be clear. Diagnostic or exploratory analysis of a classification will often be improved by ignoring large parts of the classification. It may be better to ignore too much than too little of the classification, provided one is left with enough information at the end to construct diagnostic measures for the full table. In the present case, did I not wish to show the evolution of the array of mobility ratios under successively improved specifications, I would specify Models Q1, Q2, and Q3 in advance and look at the mobility ratios only under Model Q3--because it fits well--as a guide to specification of a more parsimonious model. In fact, after grouping cells with similar mobility ratios under Model Q3, I wrote the specification in Table 2 by inspection.

In this context it is instructive to recall my earlier comparison of constraints on parameters of the saturated model with constraints on parameters in my multiplicative model. In the former case parameters for row-by-column interaction are constrained within each row and column of the table; that is, the sum of interaction parameters is zero (in the loglinear model) and the product of interaction parameters is unity (in the multiplicative model) within each row and within each column of the table. In the latter case a similar constraint holds over all cells of the classification, but not within each row and column. The quasi-independence model, like our multiplicative model, provides superior diagnostic insights because it, too, does not constrain interaction parameters within rows or columns. This is easy to see if we consider the array of expected frequencies under the quasi-independence model. Within the zone of quasi-independence there are no row-by-column interactions. In the remainder of the table the observed frequencies are

fitted exactly with multiplicative parameters defined by the ratio of observed frequencies to those expected from row and column effects within the quasi-independent zone of the table. Given any set of expected frequencies (and row and column effects) in the quasi-independent zone of the table, the remaining frequencies (and their corresponding interaction parameters) can vary freely, so clearly there are no constraints on the interaction parameters within rows or columns. The absence of row and column constraints on interaction parameters, shared by my model and by quasi-independence models, leads in both instances to improved diagnostic and interpretative insights into the mobility table.

Model Specification by Median Fitting

Throughout this exposition I have relied on maximum-likelihood estimates obtained by iterative proportional rescaling, but computationally simpler methods may suffice, especially in the analysis of small tables. To illustrate, I repeat the estimation of Model Q3 by median fitting. For a full treatment of such methods, see Tukey (1977: Ch. 11). Panel A of Table 17 gives the natural logs of observed frequencies in those cells of Table 1 which are quasi-independent under Model Q3. In the last column I show row means obtained as the first step in the analysis. Deviations of the entries in Panel A from the row means are carried forward to Panel B, and in the last row of Panel B I take column means. Panel C shows deviations from the column means in Panel B, and here I begin a similar analysis by medians. In Panels C though I, I alternate the extraction of row and column medians until, at Panel I, the row medians are each rather close to zero. In rows or columns containing an even number of cells I take the midpoint of the two central observations as the median. The solution requires minimal calculation because most of the medians

can be ascertained by inspection; beyond the initial computation of means subtraction is the main arithmetic operation.

TABLE 17 ABOUT HERE

In Table 18 I cumulate the several row effects and the several column effects obtained in Table 17. The resultant row and column sums are the estimates of main effects under Model Q3. The row effects are larger (absolutely) than the column effects because they include the grand mean of the observations. In Panel A of Table 19 I use the row and column effects to estimate logs of frequencies in the cells of Table 1 which were ignored in fitting Model Q3. For example, the estimate in cell (1,1) is 5.219 + .337 = 5.556. In Panel B of Table 19 I assemble the residuals from the interior cells in Panel I of Table 17 together with the deviations of observed from expected log frequencies in the cells ignored in fitting the model. Panel C shows the antilogs of the entries in Panel B, which can be interpreted as the mobility ratios in Table 15 under Model Q3. Obviously, my crude manual fit does not exactly reproduce the mobility ratios obtained from the maximum-likelihood estimates, but the pattern is close enough for diagnostic purposes.[12]

TABLES 18 AND 19 ABOUT HERE

In using exploratory methods, like those illustrated here, one always runs the risk of overfitting data; that is, one may model features of the data which occur only as a result of sampling fluctuation. The surest protection against overfitting is independent validation, the test of a model against independent observations. It may also help to smooth the data statistically, e.g., by averaging across population subgroups or by ---ing some categories of the row and column classifications. I have

illustrated these methods elsewhere (Hauser 1979), and I shall not elaborate them here. Rather, it may be more instructive to consider other data to which the present methods may fruitfully be applied.

Sibling Resemblance in Educational Attainment

In the Wisconsin Longitudinal Study of Social and Psychological Factors in Achievement (Sewell and Hauser 1980) more than 9,000 male and female high school graduates were interviewed in 1975--18 years after high school graduation--and in roughly 2,000 cases a randomly selected sibling was interviewed in 1977.[13] Panel A of Table 20 gives the counts in a classification of self-reports of educational attainment by respondents (in the 1975 study) and their siblings (in 1977). The association between these two variables should be indicative of the strength and manner in which social, psychological, and genetic factors in the family of orientation lead to similarity in the educational attainments of siblings. There is an inherent symmetry in the way we look at this table. Neither sibling's education is causally prior to that of the other; rather, we think of both as determined by exogeneous factors in their families and in the larger society. In looking at these data, our main interest lies in the joint occurrence of each pair of educational outcomes, and we want to abstract the density of each joint occurrence from the widely varying relative frequencies of levels of schooling.

By inspection of Panel A of Table 20, it appears that the symmetry in our interpretation of the table is reflected in the counts. Notice that corresponding row and column sums are very similar, as are corresponding entries, x_{ij} and x_{ji}, across the main diagonal. This symmetry appears also in corresponding row and column percentage distributions, shown in Panel B and Panel C of Table 20. Formally, the hypothesis of symmetry says $m_{ij} = m_{ji}$ for all $i \neq j$; the maximum-likelihood estimates of off-diagonal frequencies

are just $\hat{m}_{ij} = \hat{m}_{ji} = (x_{ij} + x_{ji})/2$. Diagonal frequencies are not constrained by this model, so $\hat{m}_{ii} = x_{ii}$ for all i. Under this hypothesis the test statistic G^2 is distributed as X^2 with K(K-1)/2 df (Bishop, Fienberg and Holland 1975: Ch. 8).

TABLE 20 ABOUT HERE

Panel D of Table 20 gives the maximum-likelihood estimates; the model fits rather well, for $G^2 = 11.59$ with 15 df. Panel E gives the row percentage distributions of the symmetric counts; these are of course the same as corresponding column percentage distributions. Most persons complete only 12 years of schooling, but there is a secondary mode at 16 years--the completion of a 4-year college. Educational attainments of siblings are positively correlated, but even among pairs where one sibling attended graduate school, the chances were about equal--30 percent--that the other sibling only graduated from high school or only graduated from college.

While the finding of symmetry greatly simplifies the interpretation of the data, one might hope to find yet a more parsimonious pattern of interaction underlying the observations. Panels F, G, and H of Table 20 show the values of three of the indexes of interaction that were discussed above (computed from counts in Panel A). As shown in Panel F, the model of simple independence fits poorly. The ratios of observed to estimated frequencies suggest little tendency for siblings to share in completion of grades 12 or 13, but there is a stronger tendency toward joint completion of grades 14 and grades 16 or 17+. Further, the ratios are curiously high in cells (13, 15) and (15, 13). Panel G gives parameters of the saturated model. The parameter in cell (12, 12) is now the largest in the table; this shows how the corresponding entry in Panel F was depressed by the large relative frequencies of high school graduation among respondents and siblings. There remain tendencies toward

joint completion of grades 14 and 16 or 17+, but these are not so large as in Panel F. Again, the relatively large entries in cells (13, 15) and (15, 13) are surprising; those two levels of schooling are neither adjacent, nor are they typical points of termination in the educational process. Panel H shows counts adjusted to uniform row and column sums of six, and these show substantially the same pattern as the parameters in Panel G. All three of these sets of indices show a general tendency for the values to fall as one moves away from the main diagonal (again, excepting cells (13, 15) and (15, 13).

Panel I specifies the interaction parameters in a very simple multiplicative model which is based upon some--but not all of our observations about Panels F, G, and H. The model says that there are equal tendencies of siblings to share in the completion of grades 12, 14, and 16 or 17+. These are the major termination points of schooling in the United States. Otherwise, the model says, there are no tendencies toward association or dissociation between the educational attainments of siblings. This model fits the data of Panel A rather well, yielding $G^2 = 22.59$ with 24 df; the model uses only 1 df for interaction. As shown beneath Panel J of Table 20, the estimates say that sibling pairs are 2.84 times more likely to complete 12, 14, or 16 and 17+ years of school s to complete any other combination of years of schooling. Panel J displays the products (R^*_{ij}) of parameter estimates and residuals under this model. These generally confirm our inference about the fit of the model, but there may be a slightly lower tendency than estimated for siblings to cluster at grade 14 and a slightly higher tendency for them to cluster at grades 17+. In any event, the overriding feature of the data--which is not accessible by inspection--is the tendency for siblings to share in the completion of major segments of the educational process. Otherwise,

there is little or no tendency toward similarity in their educational attainments.

The data of Table 20 are fitted well, also, by a model that incorporates two further simplifications: (1) that the main effects are identical in corresponding rows and columns, and (2) that the joint density of siblings' attainments at major educational transitions is precisely three times greater than densities elsewhere in the table.

The first of these two hypotheses--marginal homogeneity--is suggested by the earlier finding that the counts are symmetric. Symmetry incorporates two distinct hypotheses. First, it says that interaction effects are equal in corresponding cells above and below the main diagonal, so for example, $\delta_{ij} = \delta_{ji}$ for all i and j in the model of equation 1 and $u_{12(ij)} = u_{12(ji)}$ for all i and j in the model of equation 26. This hypothesis--called quasi-symmetry--is already implicit in the specification of parameters in Panel I of Table 20. Second, symmetry also implies that main effects are equal in corresponding rows and column, that is, row and column marginal distributions are homogeneous. As noted earlier, the combination of quasi-symmetry and marginal homogeneity implies equality in population counts in corresponding cells above and below the main diagonal, that is, $m_{ij} = m_{ji}$ for all i and j.

Models of symmetry, quasi-symmetry, and marginal homogeneity can easily be fitted and evaluated using a 3-way array composed of the original table and its transpose. Let 1 = the original row variable, 2 = the original column variable, and 3 = the transposition variable (whose values specify the original table and its transpose). Symmetry is imposed by fitting the marginal configurations (12)(3), leaving K(K-1)/2 degrees of freedom for error. Quasi-symmetry is imposed by fitting the marginal configurations (12)(13)(23), leaving (K-1)(K-2)/2 degrees of freedom for error. The goodness-of-fit test statistics [illegible]ed by this set-up should be divided by two because of the double entry

of each count; equivalently, each count in the array may be divided by two. The difference between the test statistics under symmetry and quasi-symmetry yields a test of the hypothesis of marginal homogeneity with K-1 degrees of freedom, one for each distinct constraint on row and column effects.

For example, in the data of Panel A of Table 20 we have already noted that symmetry yields a likelihood-ratio test statistic $G^2 = 11.59$ with 15 df. Under quasi-symmetry, we obtain $G^2 = 8.95$ with 10 df, so the test of marginal homogeneity yields $G^2 = 2.64$ with 5 df. Thus, the data are consistent with both of the hypotheses subsumed under symmetry.

The second simplification is suggested by the model of Panels I and J of Table 20; the joint density of observations is roughly three times greater in cells (12,12), (14,14), (16,16), (16,17), (17,16), and (17,17) than elsewhere in the classification. I specify a model in which the density is precisely three times greater in the selected cells by fitting only the row and column marginal configurations (as under simple statistical independence) to a table of starting values in which 3s have been entered in the high density cells and 1s in all other cells. In order to estimate the simplified model--incorporating symmetry and three-fold density--I modified the starting values in the set-up used to test symmetry and quasi-symmetry and fitted just the univariate marginal distributions for rows, columns, and transposition. This simplified model fits the data very well; $G^2 = 25.99$ with 30 df. That is, in the simplified model I estimate only six parameters, one for the scale factor (total count) and five for the row/column effects. Finally, on the basis of this analysis the test statistic may be partitioned into four additive components: (1) departures from quasi-symmetry ($G^2 = 8.95$ with 10 df), (2) other departures from the model of differential density at major schooling transitions ($G^2 = 13.64$ with 14 df), (3) departures from marginal homogeneity ($G^2 = 2.64$ with 5 df), and (4)

departures from three-fold density (G^2 = .76 with 1 df). None of these components of error approaches statistical significance.

Occupational Similarity of Friends

In Table 21, Panel A shows counts in a classification of the occupations of a sample of Detroit men and their friends.[14] The data were obtained from a sample of 1,000 men who were asked to name and to describe the occupations of their three best friends. Since the sample clusters friends within respondents, there are in effect fewer than the 2,873 nominal observations suggested by the sum of counts, but I have made no correction for this lack of independence.

As shown in Panel B, respondents choose friends whose occupations resemble their own. Between 41 and 48 percent of the nominations from each occupational group fall within the same group. Panel C shows the ratios of observed frequencies to those estimated under simple independence. In the source, Jackson (1977:63) refers to the diagonal entries of Panel C as

TABLE 21 ABOUT HERE

self-selection ratios; he comments that self-selection is highest among upper white collar men and least among blue collar men, while friendship declines with social distance throughout the table. As shown in Panel C, the model of simple independence fits these data very poorly (G^2 = 778.17 with 9 df), and for this reason the ratios in Panel C are quite misleading.

One might expect to find a certain symmetry in the table, for it is difficult to see how affinity between occupational categories as such should depend on the direction of choice. At the same time the counts in Panel A of Table 21 are clearly not symmetric, for men at the top of the occupational hierarchy must choose friends of the same of lower status, while those at

the bottom must choose friends of the same or higher status. The model of symmetry, which fit in the previous case, must be rejected here (G^2 = 42.46 with 6 df). However, a weaker model, quasi-symmetry (Bishop, Fienberg and Holland 1975: Ch. 8), does fit these data (G^2 = 2.25 with 3 df). The model of quasi-symmetry, like that of symmetry, says interaction effects are the same in corresponding cells above and below the diagonal, but it does not add the constraint of homogeneity in corresponding marginal effects. Thus, under quasi-symmetry m_{ij} may differ from m_{ji} because the i^{th} row and the i^{th} column effects may differ and because the j^{th} row and the j^{th} column effects may differ. Panel D of Table 21 gives the estimated frequencies under quasi-symmetry, and these were used in the next stage of model selection.

Since theory and data suggest that observations cluster along the diagonal, I fitted a quasi-independence model that ignored those four cells. The starting values for that model are shown in Panel E and, following the methods used in Table 15, ratios of observed to expected frequencies are given in Panel F. The model does not fit well, but it is a great improvement over simple independence. The new self-selection ratios suggest that within-group choice is high at either extreme of the status hierarchy. Moreover, when out-group choice occurs, white collar men choose other (higher or lower status) white collar men, and to about the same degree blue collar men choose other (higher or lower status) blue collar men. White collar men are much less likely to choose blue collar friends, and vice versa. When the blue-collar/white-collar line is crossed there is little status differentiation in choice within the out-group.

Panel G of Table 21 specifies a model with the features which appeared in the ratios in Panel F, and Panel H displays the relative densities (R^*_{ij})

under this model in the original table (Panel A). The model of Panel G fits very well (G^2 = 6.46 with 5 df); moreover, a very simple pattern appears in the parameter estimates under this model (shown beneath Panel H), namely, that the estimates are in the ratios 1:2:3:4:5. With this as a clue, in the final model, I specified that relative densities would have the hypothesized values by fitting the row and column marginals of the observed table to the starting values shown in Panel I of Table 21. Note that the entries in Panels G and I are identical, but the latter are hypothesized parameter values, and the former are arbitrary subscripts of variables. Since the parameters are specified as constants, the final model (nominally) has 9 df, and the fit is very good (G^2 = 7.63); little is lost by imposing the constraint of linearity on the parameters. Plainly, my final model of the Detroit data differs substantially from Jackson's analysis by inspection of departures from independence. There are different tendencies toward self-selection in each occupational gro p, and these are greatest at the extremes of the status hierarchy. When out-group friendship occurs, it tends to be among white collar men (for white-collar choosers) and among blue-collar men (for blue collar choosers). White collar men rarely choose blue-collar friends, and vice versa; when such choice occurs, status distinctions within the out-group are ignored. This final model is just as parsimonious as that in the source; it estimates no parameters for interaction. Yet, unlike Jackson's analysis, it fits the data well.

Comparisons Between Classifications

In a series of analyses my colleagues and I were able to locate no sociologically interpretable historical changes in the relative occupational mobility chances of American men when we used saturated models of the mobility ...ication (Hauser et al. 1975b, Featherman and Hauser 1978: Ch. 3).

Reviewing these null findings we thought the lack of significant evidence of change might be the low statistical power of our models, rather than the absence of change in mobility chances. A parsimonious model increases statistical power in comparisons; the need for parsimony, as much as the issue of interpretation, motivates our efforts to apply models of the present form. While other models may be even more parsimonious, my models do use far fewer parameters to fit a given table than does the saturated model. In discussing comparative methods, I start with general hypotheses about the restrictions imposed by the model and later take up more specific hypotheses about the values of parameters or sets of parameters.

If we borrow a model that fits one classification in order to fit a second classification, we readily obtain an explicit test of the partition of cells in the initial model. Recall, for example, that the model of Table 2 fits the aggregate table of mobility from father's occupation to son's first occupation in the 1973 OCG survey with G^2 = 66.5 on 12 df. To test the assignment of cells to levels in this model, I use the same model in an analysis of mobility from father's occupation to son's first occupation by age using data from the 1962 OCG survey. That is, where P = father's occupational stratum, W = occupational stratum of son's first job, A = age in 5-year groups, and H = the model of Table 2, I fit the model (PA)(WA)(HA) to the 1962 data and obtain a test statistic of G^2 = 121.2 with 108 df, which is not statistically significant. That is, conditional on variation in occupational origins and destinations between cohorts, the same set of equality restrictions fits interactions between father's occupation and son's first occupation in the 1962 OCG survey as in the 1973 OCG survey. The lack of significant departures from this model does not indicate that mobility chances are numerically identical (or even remotely similar) in these two surveys.[15] I have tested only the hypothesis that the restrictions on

interactions across cells of the classification are met in both sets of data, not the hypothesis that interaction parameters take on the same value.

In a straightforward way we may also test the equality of interaction parameters among two or more cross-classifications. For example, I compare mobility from father's occupation to son's first occupation across nine five-year age cohorts covered in the 1973 OCG survey. Under the assumption that mortality is negligible during the prime working ages (20 to 64), while first jobs pertain to a fixed point in the life-cycle, these comparisons reflect conditions of labor market entry during the period from the late 1920s to the early 1970s. As shown in the first column at the top of Table 22, I begin by fitting the model (PA)(WA)(H), in which occupational origins and destinations vary across cohorts, but relative mobility chances do not vary. I have 9 mobility subtables, each with 16 degrees of freedom after conditioning on the observed marginal distributions; since the five-level model of Table 4 uses just four degrees of freedom, there are 140 degrees of freedom for error. Under this specification I obtain the significant test statistic, G^2 = 235.3. At the same time the simple model of Table 2 is quite powerful; it explains G^2 = 5667.7 with 4 df (Featherman and Hauser 1978:200). I also fit the same table with the model (PA)(WA)(HA), which fits origin and destination effects as in the initial model, but permits the parameters of the model to vary across cohorts. Under this model I obtain G^2 = 175.6 with 108 df; relative to the initial model, I fit four more parameters for each of eight subtables. Again, the test statistic is statistically significant, showing that there are non-chance departures from the specification within one or more cohorts. More important, since the model (PA)(WA)(HA) is obtained from (PA)(WA)(H) by relaxing restrictions on interactions in the latter, I may test these restrictions by taking the difference between the two test statistics. I obtain G^2 = 59.7 with 32 df (top line, third column of Table 22),

whose probability is very small under the null hypothesis. Thus, there are statistically significant intercohort variations in parameters of the model of Table 2. In passing I note that the last test statistic is larger relative to its degrees of freedom than the test statistic for intercohort change under the model (PA)(WA)(PW) in the same classification: G^2 = 166.6 with 128 df, for which α = .012.

TABLE 22 ABOUT HERE

To summarize, by conditioning on the marginal distributions and fitting the models of constant interaction and of variable interaction, subject to a given design matrix, we can test the fit of each of those two models; further, a contrast between those two models yields a global test of change in the parameters of the design matrix. Using a similar procedure we can test hypotheses about change in each level parameter of a cross-classification, and at the same time we can test hypotheses about the lack of fit within each level of the design matrix. Again, we construct appropriate test statistics by contrasting hierarchical models and exploiting the additive properties of the likelihood-ratio test statistic (G^2); see Bishop, Fienberg and Holland (1975:126-127).

Figure 2 defines and shows the relationships among four types of models of the cross-classification from which we obtain the desired test statistics. Consider a four-way incomplete classification in which two factors with I and J categories, respectively, represent the mobility table (or other cross-classification), a third factor with K categories specifies the interaction effects, and a fourth factor with L categories represents the replicates of the I x J cross-classification which we wish to compare. Further, denote

the number of (independently variable) cells within the k^{th} level of the design matrix by M_k.

FIGURE 2 ABOUT HERE

As shown in the first row of Figure 2 I condition on the observed marginal distributions of the I x J classification and fit models of constant association and of variable association to the entries in the four-way classification. Under the model of constant association, e.g., (PA)(WA)(H) in the example of Table 22, I denote the likelihood ratio test statistic by G_a^2, and under the model of variable association, e.g., (PA)(WA)(HA) in the example of Table 22, I denote the likelihood ratio test statistic by G_b^2. Then, as shown in the second row of Figure 2, I blank out or ignore (by entering structural zeros) each of the counts at the k^{th} level of the design matrix, and I fit the models of constant association and of variable association to the counts in the truncated four-way classification. I repeat this procedure for each of the K levels of the design matrix. Under the model of constant association in the truncated classification I denote the likelihood ratio test statistic by G_c^2, and under the model of variable association in the truncated classification I denote the test statistic by G_d^2. When I blank out or ignore entries at the k^{th} level of the I x J classifications I am implicitly fitting constants to each of those entries, so the degrees of freedom under models of the truncated classification are reduced by the number of (independently variable) counts at the k^{th} level of each I x J classification.[16]

By comparing the terms G_a^2, G_b^2, G_c^2, and G_d^2, I can test a variety of hypotheses about change in interactions (level parameters) among the several I x J classifications and about lack of fit within levels of the model.

These contrasts and the associated degrees of freedom are shown in Figure 3. I have already applied (in Table 22) a contrast like that on Line 1, which gives an overall test of constancy in the level parameters across the several I x J classifications. Recall that the model of variable association relaxes restrictions on parameters of the model of constant association, so the comparison, $G_a^2 - G_b^2$, yields a test of constancy in all K level parameters across the L mobility classifications. Since a K-level design has K - 1 df, while the model of constant association specifies only one set of level parameters, it takes (K - 1)(L - 1) more degrees of freedom to specify the model of variable association than to specify the model of constant association.

FIGURE 3 ABOUT HERE

The contrasts on Lines 2 and 3 of Figure 3 are of lesser interest, but I include them for the sake of completeness. The comparison on Line 2, $G_c^2 - G_d^2$, is analogous to that on the first line, but the test ignores parameters at the k^{th} level, which have been excluded both from the models of constant and of variable association; thus, the test has (L - 1) fewer degrees of freedom than the global test on Line 1. In the model of constant association the L sets of counts at the k^{th} level of the I x J classification are fitted by a single parameter, but (implicitly) a parameter is fitted to each of these counts when entries at the k^{th} level are blocked. Thus, the comparison on Line 3 of Figure 3, $G_a^2 - G_c^2$, yields a test of the combined effects of change and of lack of fit at the k^{th} level of the design matrix, and this test has LM_k - 1 degrees of freedom.

The comparison of Line 4 of Figure 3, $G_b^2 - G_d^2$, yields a test of fit at the k^{th} level of the design matrix across the L mobility classifications.

G_b^2 is obtained under the model of variable association in the full classification, so it reflects lack of fit at each of the K levels across the L mobility classifications. I obtain G_d^2 by relaxing the $M_k - 1$ equality restrictions on interactions at the k^{th} level within each of the L mobility classifications, so the test statistic has a total of $L(M_k - 1)$ degrees of freedom. Since the models used to generate this test condition both on the marginal distributions and interaction parameters of the L distinct I x J classifications, the overall comparison, $G_b^2 - G_d^2$, may be regarded as the sum of L independent test statistics, each with $M_k - 1$ degrees of freedom, pertaining to lack of fit within the k^{th} level of one of the L classifications. Plainly, an analogous test may be obtained at the k^{th} level of any I x J classification merely by contrasting the fit of the full classification with that of the classification from which counts at the k^{th} level have been removed.

In the fifth line of Figure 3 I show a comparison of all four of the test statistics from Figure 2 that tests the L - 1 equality restrictions imposed by the model of constant association at the k^{th} level of the design. One may think of this comparison as a contrast between the models of Lines 1 and 2 of Figure 3, or alternatively, one may think of it as a contrast between the models of Lines 3 and 4 of Figure 3. That is, the contrast in Line 2 differs from that in Line 1 only in permitting the k^{th} level parameter to vary across the L classifications, and the contrast in Line 4 differs from that in Line 3 only in permitting the k^{th} level parameter to vary across the L classifications.

I illustrate some of the contrasts in Figure 3 by continuing my analysis of intercohort change in mobility to first occupations among American men. In Lines 1 to 5 of the first panel of Table 22 I report test statistics and degrees of freedom under the model of constant association as each level of

the design (Table 2) is ignored in turn. Since levels 1, 2, and 3 each include only 1 count per cohort, I use 8 df more than in the constant association model for the full table when I ignore the entries at these three levels. There are five entries in each of the nine cohort mobility tables at level 4. When these 45 entries are ignored, the remaining row, column, and level effects are each still identified, so I lose a total of 44 degrees of freedom relative to the model of constant association in the full table.

If I used the same rule to calculate the degrees of freedom lost in ignoring level 5 as in ignoring level 4, I would end up with a negative number in Line 5, rather than the 24 degrees of freedom reported there. That is, there are 17 counts in each cohort mobility table at level 5 of the design, so one might (incorrectly) say there are 140 - [(17)(9) - 1] = -12 degrees of freedom under the model of constant association when level 5 is ignored. The error in this calculation becomes obvious when we inspect the display in Table 2. First, since level 5 covers all of Row 3 and all of Column 3 of the mobility classification, there are only four independent entries in each of Row 3 and Column 3. Second, by the same token there is no effect of Row 3 nor of Column 3 when the fifth level of the mobility classification is ignored. Thus, the erroneous calculation has subtracted four too many degrees of freedom for each of the nine intercohort tables. When we add these back in, we obtain the correct 24 degrees of freedom under constant association when level 5 is ignored.

Alternatively, we may obtain the correct degrees of freedom by enumerating the number of parameters fitted to the truncated classification. Ignoring the cells at level 5, consider the design in Table 2. For each cohort there are two separate 2 x 2 subtables which have neither rows nor

columns in common. For each cohort it takes four degrees of freedom to fit the lower right hand 2 x 2 subtable, which has, of course, just one degree of freedom for interaction. We use four degrees of freedom, also, in fitting the upper left hand 2 x 2 subtable. Since three level parameters appear in the upper left hand subtable, it may appear that the model for that subtable is underidentified. That is not the case because one of the interaction parameters (from level 4) is determined in the lower right hand 2 x 2 subtable. In summary, I fit each truncated cohort table exactly (exhausting the eight degrees of freedom of the 8 counts) with a model in which four interaction levels appear in the two 2 x 2 subtables. This explains why there are no degrees of freedom for error when I fit the model of variable association to the classification from which level 5 has been deleted. When I fit only one set of level parameters to the truncated classification under the model of constant interaction, I have (4 - 1) x 8 = 24 degrees of freedom for error, which is the same result obtained above by subtraction.

In the second part of Table 22 I show values of G^2 and degrees of freedom under the model of variable association as each level of the design (Table 2) is ignored in turn. In the full table the model of variable association leaves 108 degrees of freedom for error. Since levels 1, 2, and 3 of the design each includes only 1 count per cohort, the fit does not improve, nor do the degrees of freedom change, when I ignore the cells at those levels under the model of variable association. Since level 4 has five (independently variable) counts in each of the nine cohorts, and one degree of freedom is used at this level for each cohort by the model of variable association, I lose four degrees of freedom for each cohort when I ignore level 4 in fitting the model of variable association; this leave 72 degrees of freedom for error. As explained above, the degrees of freedom are exhausted when e level 5 in fitting the model of variable association.

Recall from Figure 3 that the contrast between the test statistic for the full table under the model of variable association (G^2 = 175.6 with 108 df) and the test statistics below it in the second panel of Table 22 are of the form, $G_b^2 - G_d^2$. That is, they reflect lack of fit at each level of the design. Since there is only one count at each of levels 1, 2, and 3, there is no lack of fit at these levels under the model of variable association; this is, of course, a formal property of the design in Table 2, and it has no confirmatory value. When I compare the test statistics in Lines 1 and 4, I obtain G^2 = 34.8 with 36 degrees of freedom. Since the test statistic is less than its expected value, I infer that the fit is satisfactory at level 4 of the design. There are no degrees of freedom for error when I ignore level 5 of the design, so I may attribute all of the lack of fit (G^2 = 175.6 with 108 df) to level 5. Obviously, this test statistic is significant, and the result suggests I might refine the present design by splitting level 5 in Table 2 into two or more levels. Note these tests for lack of fit (and the companion tests for change reported later) are not independent; there is no inconsistency between my use of the overall test statistic for error to evaluate the fit at level 5 and a component of the same statistic to evaluate the fit at level 4.

In the third panel of Table 22 I report on each line the differences between the entries on that line in the first and second panels. On the first line the difference is a test statistic of the form, $G_a^2 - G_b^2$, and on the remaining lines the differences are test statistics of the form, $G_c^2 - G_d^2$. On the first line I obtain the overall test statistic for change in level parameters, and the remaining lines give the test statistics for change at all but the specified level of the design matrix. Since the full cross-classification has K = 5 levels of interaction, and I compare L = 9 cohort

tables, each of the latter test statistics has $(L - 1)(K - 2) = 8 \times 3 = 24$ degrees of freedom. To obtain the test statistics for change at each level of the design, I again take differences between the test statistic for the full table and that on each line below it (recall Line 5 of Figure 3). These are reported in the fourth panel of Table 22. Intercohort changes in parameters at levels 1 and 3 are plainly not significant, while that at level 2 is (on my reading) of borderline statistical significance. At levels 4 and 5 there is statistically significant change in the parameters, whether I evaluate the nominal probabilities associated with the values of G^2 or use an appropriate simultaneous inferential procedure (Goodman 1969a).[17] Again, these tests for change are not statistically independent; note that the sum of the test statistics for the five tests is larger than the test statistic for the global contrast (59.7 with 32 df), and the sum of degrees of freedom in the five tests is 40, rather than the 32 degrees of freedom for change in the level parameters.

NOTES

[1] I assume the familiarity of the reader with loglinear models for frequency data. Fienberg (1970a, 1977), Goodman (1972a, 1972b) and Davis (1974) give useful introductions, as does the comprehensive treatise by Bishop, Fienberg and Holland (1975). I rely heavily on methods for the analysis of incomplete tables, which have been developed by Goodman (1963, 1965, 1968, 1969a, 1969b, 1971, 1972c), Bishop and Fienberg (1969), Fienberg (1970b, 1972), and Mantel (1970); again, Bishop, Fienberg and Holland (1975, especially pp. 206-211, 225-228, 282-309, 320-324) is valuable. Applications of loglinear models to occupational mobility data include several of the papers by Goodman just cited and, also, Hope (1974, 1980), Hauser et al. (1975b). Pullum (1975), Iutaka et al. (1975), Featherman, Jones and Hauser (1975), Ramsøy (1977), Hauser and Featherman (1977), Baron (1980), Hauser (1978), Goldthorpe, Payne and Llewellyn (1978), Goldthorpe and Payne (1980), Duncan (1979), and Featherman and Hauser (1978).

[2] In the I x J cross-classification there are (I - 1) (J - 1) degrees of freedom for two-way interaction. The conventional structural model yields two-way interaction effects for each of I x J counts by constraining the product of interaction effects within each row and within each column of the table; these constraints identify (I - 1) (J - 1) independent interaction effects. Instead, the model of equation 1 identifies the two-way interaction effects by constraining some of them to be equal across cells of the classification.

[3]Many of these models--as well as problems in comparing their goodness of fit--are reviewed by Bishop, Fienberg and Holland (1975): Chs. 5, 8, 9), and some of the same models are discussed by Haberman (1974: Ch. 6). Duncan (1979) and Goodman (1979b) have recently proposed additional models for classifications of ordered categorical data.

[4]I denote parameters of the loglinear model in equation 2 by u^*, $u^*_{1(i)}$, etc., in order to distinguish them from the u-terms of the conventional row-by-column parameterization (Bishop, Fienberg, and Holland 1975: Ch. 2).

[5]The reported frequencies are based on a complex sampling design and have been weighted to estimate population counts while compensating for certain types of survey nonresponse. The estimated population counts have been scaled down to reflect underlying sample frequencies, and a further downward adjustment was made to compensate for departures of the sampling design from a simple random sampling. The frequency estimates in Table 1 have been rounded to the nearest integer, but my computations are based on unrounded figures. I treat the adjusted frequencies as if they had been obtained under simple random sampling (Featherman and Hauser 1978:App.B).

Occupation, industry and class of worker were coded in detail using classification methods of the 1960 U.S. Census, and the detailed occupation codes were aggregated to form the broad groups shown in the table. The broad occupation groups have been defined in a slightly unconventional way (on the basis of data on the schooling and incomes of current occupational incumbents). Sales workers other than retail sales workers have been

placed in the upper nonmanual group, while proprietors have been placed in the lower nonmanual group. This does not substantially affect the findings relative to those based on a more conventional classification of upper and lower white-collar jobs (Featherman and Hauser, 1978:180-184).

[6]This observation is elaborated by Featherman and Hauser (1978:177-180).

[7]Unsubscripted e is the base of natural logarithms and should not be confused with the sample residuals in the multiplicative model, $e_{ij} = x_{ij}/\hat{m}_{ij}$.

[8]Larntz (1978) has shown that z_{ij} has better small-sample properties than do components of G^2 (the likelihood-ratio test statistic) or Freeman-Tukey deviates.

[9]See Featherman and Hauser (1978:66-67) for further discussion of these patterns.

[10]Just as I have shown that parameters of the saturated model are mixtures of parameters of my model, it is also easy to show that the Mosteller adjustments yield indexes which are mixtures of the parameters of my model.

[11]In more complex models other methods may be needed to estimate the missing frequencies, such as those used to estimate parameters for models of the full table. The manual computations are often convenient, and ECTA converges more rapidly when cells with unique parameters are

ignored than when the program is forced to fit them exactly.

[12] It took the author about an hour to prepare this illustration using a small electronic calculator.

[13] This example was developed by Brian Clarridge. Both the subsamples of respondents and of their siblings were highly stratified by sex and educational attainment, but I have treated the data as if they were obtained by simple random sampling.

[14] This example was developed with the assistance of Shu-Ling Tsui. The Detroit data were collected by Edward Laumann; I estimated the counts from percentages and marginal frequencies in a secondary analysis of these data by R. Jackson (1977). For this reason the counts in Panel A may differ slightly from those in the original data.

[15] There are methodological differences in the measurement of first occupations in these two surveys, and for this reason it would not be surprising if the two models (or their parameters) differed substantially. For more details about this comparison see Featherman and Hauser (1978: 200-208).

[16] The degrees of freedom of the statistics G_c^2 and G_d^2 may be difficult to enumerate because they depend on the pattern of the design matrix as well as on the number of cells at the k^{th} level. For example, when level 5 of the design in Table 2 is blanked out, we delete only 4 independent entries in each of the third row and the third column, and we no longer need to estimate marginal effects of the third row or the third column of the mobility classification. Thus, one is well-advised to apply the general rule that the degrees of freedom for error under a model is equal to the number of cells in the classification less the number of independent constants fitted to it.

[17] See Featherman and Hauser (1978:200-208) for further discussion of these results.

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TABLE 1

Counts in a classification of mobility from Father's (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation: U.S. Men Aged 20-64 in 1973

Father's occupation	Son's occupation: Upper Nonmanual	Lower Nonmanual	Upper Manual	Lower Manual	Farm	Total
Upper Nonmanual	1414	521	302	643	40	2920
Lower Nonmanual	724	524	254	703	48	2253
Upper Manual	798	648	856	1676	108	4086
Lower Manual	756	914	771	3325	237	6003
Farm	409	357	441	1611	1832	4650
Total	4101	2964	2624	7958	2265	19,912

NOTE: Counts are based on observations weighted to estimate population counts and compensate for departures of the sampling design from simple random sampling. Broad occupation groups are upper nonmanual: professional and kindred workers, managers and officials, and non-retail sales workers; lower nonmanual: proprietors, clerical and kindred workers, and retail salesworkers; upper manual: craftsmen, foremen and kindred workers; lower manual: service workers, operatives and kindred workers, and laborers, except farm; farm: farmers and farm managers, farm laborers and foremen.

TABLE 2

Asymmetric Five-level Model of Mobility from Father's Occupation to First Full-time Civilian Occupation

Father's occupation	Son's occupation: 1	2	3	4	5
1. Upper Nonmanual	2	4	5	5	5
2. Lower Nonmanual	3	4	5	5	5
3. Upper Manual	5	5	5	5	5
4. Lower Manual	5	5	5	4	4
5. Farm	5	5	5	4	1

NOTE: Broad occupation groups are upper nonmanual: professional and kindred workers, managers and officials, and non-retail sales workers; lower nonmanual: proprietors, clerical and kindred workers, and retail sales-workers; upper manual: craftsmen, foremen and kindred workers; lower manual: service workers, operatives and kindred workers, and laborers, except farm; farm: farmers and farm managers, farm laborers and foremen.

TABLE 3

Estimated Parameters of the Model of Table 2: Mobility from Father's (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation, U.S. Men Aged 20-64 in 1973

Design factor	Category of row, column, or level (1)	(2)	(3)	(4)	(5)
Rows (father's occupation)	-.466	-.451	.495	.570	-.148
Columns (son's occupation)	.209	.190	.240	1.020	-1.660
Levels (density)	3.044	1.234	.549	.243	-.356

Grand mean = 6.277

TABLE 4

Log of Ratio of Observed to Expected Frequencies in the Model of Table 2: Mobility from Father's (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation, U.S. Men Aged 20-64 in 1973

Father's occupation	Son's occupation 1	2	3	4	5
1. Upper Nonmanual	.00*	.01	.02	-.01	-.10
2. Lower Nonmanual	.00*	.00	-.17	.06	.06
3. Upper Manual	.06	-.13	.10	-.01	-.07
4. Lower Manual	-.07	.14	-.08	.00	.04
5. Farm	.03	-.09	.08	-.01	.00*

*Fitted exactly under the model.

TABLE 5

Standardized Residuals from the Model of Table 2: Mobility from Father's (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation, U.S. Men Aged 20-64 in 1973

Father's occupation	Son's occupation				
	1	2	3	4	5
1. Upper Nonmanual	.00*	.26	.29	-.24	-.60
2. Lower Nonmanual	.00*	.04	-2.76	1.71	.45
3. Upper Manual	1.60	-3.38	2.80	-.51	-.74
4. Lower Manual	-1.97	4.14	-2.28	-.05	.60
5. Farm	.65	-1.62	1.58	-.32	.00*

*Fitted exactly under the model.

TABLE 6

New Mobility Ratios (R^*_{ij}) Under the Model of Table 2: Mobility from Father's (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation, U.S. Men Aged 20-64 in 1973

Father's occupation	Son's occupation				
	1	2	3	4	5
1. Upper Nonmanual	3.42	1.28	.71	.70	.64
2. Lower Nonmanual	1.73	1.28	.59	.75	.75
3. Upper Manual	.74	.61	.77	.69	.65
4. Lower Manual	.65	.80	.64	1.27	1.32
5. Farm	.73	.64	.76	1.27	20.91

TABLE 7

Standard Outflow and Inflow Tables: Modility from Father's (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation, U.S. Men Aged 20-64 in 1973

Father's occupation	Son's occupation (1)	(2)	(3)	(4)	(5)	Total
			Outflow			
1. Upper Nonmanual	48.4	17.8	10.3	22.0	1.4	100.0
2. Lower Nonmanual	32.1	23.3	11.3	31.2	2.1	100.0
3. Upper Manual	19.5	15.9	21.0	41.0	2.6	100.0
4. Lower Manual	12.6	15.2	12.8	55.4	4.0	100.0
5. Farm	8.8	7.7	9.5	34.6	39.4	100.0
Total	20.6	14.9	13.2	40.0	11.4	100.0
			Inflow			
1. Upper Nonmanual	34.5	17.6	11.5	8.1	1.8	14.7
2. Lower Nonmanual	17.7	17.7	9.7	8.8	2.1	11.3
3. Upper Manual	19.5	21.9	32.6	21.1	4.9	20.5
4. Lower Manual	18.4	30.8	29.4	41.6	10.5	30.1
5. Farm	10.0	12.0	16.8	20.2	80.9	23.4
Total	100.0	100.0	100.0	100.0	100.0	100.0

NOTE: Frequencies are based on observations weighted to e .imate population counts and compensate for departures of the sampling design from simple random sampling. Broad occupation groups are upper nonmanual: professional indred workers, managers and officials, and non-retail sales workers; lower nonmanual: propristors, clerical and kindred workers, and retail sales-workers; upper manual: craftsmen, foremen and kindred workers; lower manual: service workers, operatives and kindred workers, and laborers, except farm; farm: farmers and farm managers, farm laborers and foremen.

TABLE 8

...ld Mobility Ratios (R) under the Model of Simple Independence: Mobility from Father's[1)] (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation U.S. Men Aged 20-64 in 1973.

Father's occupation	Son's occupation 1	2	3	4	5
1. Upper Nonmanual	2.35	1.20	.78	.55	.12
2. Lower Nonmanual	1.56	1.56	[illegible]	.78	.19
3. Upper Manual	.95	1.06	1.59	1.03	.23
4. Lower Manual	.61	1.02	.98	1.39	0.35
5. Farm	.43	.52	.72	0.87	3.46

TABLE 9

Raw and Adjusted Standardized Deviates Under the Model of Simple Independence: Mobility from Father's (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation, U.S. Men Aged 20-64 in 1973

Father's occupation	Son's occupation (1)	(2)	(3)	(4)	(5)
	Standardized deviates				
1. Upper Nonmanual	33.13	4.14	-4.23	-15.33	-16.02
2. Lower Nonmanual	12.07	10.31	-2.49	-6.60	-12.99
3. Upper Manual	-1.50	1.60	13.68	1.08	-16.56
4. Lower Manual	-13.65	.68	-.70	18.89	-17.07
5. Farm	-17.73	-12.74	-6.94	-5.74	56.65
	Adjusted standardized deviates				
1. Upper Nonmanual	48.89	5.70	-5.71	-29.93	-21.18
2. Lower Nonmanual	17.14	13.66	-3.24	-12.39	-16.52
3. Upper Manuel	-2.38	2.37	19.82	2.27	-23.52
4. Lower Manual	-24.61	1.15	-1.15	45.05	-27.57
5. Ferm	-29.14	-19.53	-10.42	-12.47	83.40

TABLE 10

Multiplicative (τ) Parameters in a Saturated Model of Mobility from Father's (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation, U.S. Men Aged 20-64 in 1973

Father's occupation	Son's occupation 1	2	3	4	5
1. Upper Nonmanual	2.76	1.37	.96	.72	.38
2. Lower Nonmanual	1.58	1.54	.90	.88	.52
3. Upper Manual	.92	1.00	1.60	1.11	.61
4. Lower Manual	.63	1.02	1.04	1.53	.96
5. Farm	.40	.47	70	.90	8.67

Table 11

Raw and Multiplicatively Adjusted Frequencies in a Hypothetical Table

Rows	Columns 1	2	Total
A. Raw frequencies			
1	x_{11}	x_{12}	$x_{11} + x_{12}$
2	x_{21}	x_{22}	$x_{21} + x_{22}$
Total	$x_{11} + x_{21}$	$x_{12} + x_{22}$	N
B. Adjusted frequencies			
1	acx_{11}	adx_{12}	$a(cx_{11} + dx_{12})$
2	bcx_{21}	bdx_{22}	$b(cx_{21} + dx_{22})$
Total	$c(ax_{11} + bx_{21})$	$d(ax_{12} + bx_{22})$	$c(ax_{11} + bx_{21}) + d(ax_{12} + bx_{22})$

TABLE 12

Doubly Standardized Frequencies of Mobility from Father's (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation, U.S. Men Aged 20-64 in 1973

Father's occupation	Son's occupation 1	2	3	4	5	Total
1. Upper Nonmanual	[illegible]	1.18	.87	.66	.21	5.00
2. Lower Nonmanual	1.35	1.50	.92	.91	.32	5.00
3. Upper Manual	.80	1.00	1.66	1.16	.39	5.00
4. Lower Manual	.56	1.03	1.10	1.69	.62	5.00
5. Farm	.22	.29	.45	.59	3.46	5.00
Total	5.00	5.00	5.00	5.00	5.00	25.00

TABLE 13

Design Matrices for Three Models of Quasi-Independence in the 5 by 5 Mobility Table

Model	Father's occupation	Son's occupation: Partial table 1	2	3	4	5	Full table 1	2	3	4	5
Q1	1	0	1	1	1	1	2	1	1	1	1
	2	1	0	1	1	1	1	3	1	1	1
	3	1	1	0	1	1	1	1	4	1	1
	4	1	1	1	0	1	1	1	1	5	1
	5	1	1	1	1	0	1	1	1	1	6
Q2	1	0	0	1	1	1	2	3	1	1	1
	2	0	0	1	1	1	4	5	1	1	1
	3	1	1	0	1	1	1	1	6	1	1
	4	1	1	1	0	0	1	1	1	7	8
	5	1	1	1	0	0	1	1	1	9	10
Q3	1	0	0	1	1	1	2	3	1	1	1
	2	0	0	0	1	1	4	5	6	1	1
	3	1	0	0	0	1	1	7	8	9	1
	4	1	1	0	0	0	1	1	10	11	12
	5	1	1	1	0	0	1	1	1	13	14

TABLE 14

Summary of Fit of Selected Multiplicative Models: Mobility from Father's (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation, U.S. Men Aged 20-64 in 1973

Model	N*	G^2	df	Δ	G^2_H/G^2_T
Independence	19,913	6167.69	16	20.1	100.0
Q1--main diagonal blocked	11,963	683.06	11	5.5	11.1
Q2--diagonal and intra-stratum moves blocked	8,869	50.05	7	1.4	0.8
Q3--diagonal and inner diagonals blocked	5,520	15.67	3	0.6	0.3

*Sum of frequencies excluding those fitted exactly under the model.

TABLE 15

Ratios of Observed Frequencies to Estimated Frequencies at Quasi-Independent Level Under Three Models of Quasi-Independence: Mobility from Father's (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation, U.S. Men Aged 20-64 in 1973

Model	Father's occupation	Son's occupation 1	2	3	4	5
Q1	1. Upper nonmanual	3.80*	1.51	1.06	.79	.62
	2. Lower nonmanual	1.73	1.35*	.79	.77	.66
	3. Upper manual	1.06	.93	1.48*	1.02	.82
	4. Lower manual	.81	1.06	1.08	1.63*	1.45
	5. Farm	.71	.67	.99	1.28	18.13*
Q2	1. Upper nonmanual	4.94*	1.86*	1.08	.97	.93
	2. Lower nonmanual	2.48*	1.84*	.89	1.04	1.10
	3. Upper manual	1.10	.91	1.21*	1.00	.98
	4. Lower manual	.91	1.13	.96	1.74*	1.90*
	5. Farm	1.00	.89	1.11	1.70*	29.77*
Q3	1. Upper nonmanual	5.33*	1.81*	.98	1.01	1.04
	2. Lower nonmanual	2.46*	1.64*	.74*	.99	1.13
	3. Upper manual	1.01	.76*	.93*	.88*	.94
	4. Lower manual	.94	1.05	.82*	1.73*	2.03*
	5. Farm	1.10	.89	1.02	1.81*	33.92*

*Cells ignored (or fitted exactly) under the model.

TABLE 16

Expected Frequencies Under Model Q1: Mobility from Father's (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation, U.S. Men Aged 20-64 in 1973

Father's occupation	Son's occupation 1	2	3	4	5
1. Upper nonmanual	372.3*	344.1	285.6	811.5	60.1
2. Lower nonmanual	319.5	387.7*	321.8	914.5	73.3
3. Upper manual	754.9	697.7	579.0*	1645.5	131.9
4. Lower manual	934.8	864.0	717.0	2037.7*	163.3
5. Farm	578.6	534.8	443.8	1261.2	101.1*

*Cells ignored (or fitted exactly) under the model.

TABLE 17

Manual Fit of Model Q3 to Table 1

Father's occupation	Son's occupation 1	2	3	4	5	Effect
	A. Log of frequency in [illegible]ted cells					**Row mean**
1. Upper nonmanual	-	-	5.710	6.460	3.696	5.291
2. ~~Lower nonmanual~~	-	-	-	~~6.555~~	~~3.881~~	~~5.218~~
3. Upper manual	6.682	-	-	-	4.680	5.681
4. Lower manual	6.6[illegible]	6.818	-	-	-	6.724
5. Farm	6.014	5.878	6.089	-	-	5.994
	B. Data less row means					
1. Upper nonmanual	-	-	.419	1.175	-1.595	
2. Lower nonmanual	-	-	-	1.337	-1.337	
3. Upper manual	1.001	-	-	-	-1.001	
4. Lower manual	-.095	.094	-	-	-	
5. Farm	.020	-.116	.095	-	-	
Column means	.309	-.011	.257	1.256	-1.311	
	C. Data less column means					**Row median**
1. Upper nonmanual	-	-	.162	-.081	-.284	-.081
2. Lower nonmanual	-	-	-	.081	-.026	.028
3. Upper manual	.692	-	-	-	.310	.501
4. Lower manual	-.404	.105	-	-	-	-.150
5. Farm	-.289	-.105	-.162	-	-	-.162
	D. Data less row medians					
1. Upper nonmanual	-	-	.243	.000	-.203	
2. Lower nonmanual	-	-	-	.053	-.054	
3. Upper manual	.191	-	-	-	-.191	
4. Lower manual	-.254	.255	-	-	-	
5. Farm	-.127	.057	.000	-	-	
Column median	-.127	.156	.122	.027	-.191	
	E. Data less column medians					**Row median**
1. Upper nonmanual	-	-	.121	-.027	-.012	-.012
2. Lower nonmanual	-	-	-	.026	.137	.082
3. Upper manual	.318	-	-	-	0	.159
4. Lower manual	-.127	.099	-	-	-	-.014
5. Farm	0	-.099	-.122	-	-	-.099
	F. Data less row medians					
1. Upper nonmanual	-	-	.133	-.015	.000	
2. Lower nonmanual	-	-	-	-.056	.055	
3. Upper manual	.159	-	-	-	-.159	
4. Lower manual	-.113	.113	-	-	-	
5. Farm	.099	.000	-.023	-	-	
Column median	.099	.056	.055	-.036	.000	
	G. Data less column medians					**Row median**
1. Upper nonmanual	-	-	.078	.021	.000	.021
2. Lower nonmanual	-	-	-	-.020	.055	.018
3. Upper manual	.060	-	-	-	-.159	-.050
4. Lower manual	-.212	.057	-	-	-	-.078
5. Farm	.000	-.056	-.078	-	-	-.056
	H. Data less row medians					
1. Upper nonmanual	-	-	.057	.000	-.021	
2. Lower nonmanual	-	-	-	-.038	.037	
3. Upper manual	.110	-	-	-	-.109	
4. Lower manual	-.134	.133	-	-	-	
5. Farm	-.056	.000	-.022	-	-	
Column median	-.056	.068	.018	-.019	-.021	
	I. Data less column medians					**Row median**
1. Upper nonmanual	-	-	.039	.019	.000	.019
2. Lower nonmanual	-	-	-	-.019	.058	.020
3. Upper manual	.054	-	-	-	-.088	-.017
4. Lower manual	-.190	.067	-	-	-	-.062
5. Farm	0	-.068	-.040	-	-	-.040

NOTE: See text for explanation.

TABLE 18

Cumulation of Effects from Table 17

	Panel of Table 17								
Occupation	A	B	C	D	E	F	G	H	Total
				Row effects					
1. Upper nonmanual	5.291		-.081		-.012		.021		5.219
2. Lower nonmanual	5.218		.028		.082		.018		5.346
3. Upper manual	5.681		.501		.159		-.050		6.291
4. Lower manual	6.724		-.150		-.014		-.078		6.482
5. Farm	5.994		-.162		-.099		-.056		5.677
				Column effects					
1. Upper nonmanual		.309		-.127		.099		-.066	.337
2. Lower nonmanual		-.011		.156		.056		.068	.269
3. Upper manual		.257		.122		.055		.018	.452
4. Lower manual		1.256		.027		-.036		-.019	1.228
5. Farm		-1.311		-.191		.000		-.021	-1.523

TABLE 19

Summary Analysis of Model Q3 Fitted by Medians

Father's occupation	Son's occupation 1	2	3	4	5
A. Expected logs of frequencies in cells ignored in fitting					
1. Upper nonmanual	5.556	5.488	-	-	-
2. Lower nonmanual	5.683	5.615	5.798	-	-
3. Upper manual	-	6.560	6.743	7.519	-
4. Lower manual	-	-	6.934	7.710	4.959
5. Farm	-	-	-	6.905	4.154
B. Observed less expected logs of frequencies					
1. Upper nonmanual	1.698	.768	.039	.019	.000
2. Lower nonmanual	.902	.647	-.261	-.019	.058
3. Upper manual	.054	-.086	.009	-.095	-.088
4. Lower manual	-.190	.067	-.286	.399	.509
5. Farm	.000	-.068	-.040	.480	3.359
C. Antilogs of entries in panel B					
1. Upper nonmanual	5.46	2.16	1.04	1.02	1.00
2. Lower nonmanual	2.46	1.91	.77	.98	1.06
3. Upper manual	1.06	.92	1.01	.91	.92
4. Lower manual	.83	1.07	.75	1.49	1.66
5. Farm	1.00	.93	.96	1.62	28.76

TABLE 20

Respondent's Education (Years of School) by Sibling's Education: Wisconsin Sample

Respondent's Education	Sibling's Education						
	12	13	14	15	16	17+	Total
A. Frequencies							
12	881	51	53	27	109	58	1179
13	65	8	7	7	18	6	111
14	40	9	14	7	14	11	95
15	23	7	4	2	6	6	48
16	91	20	12	9	77	53	262
17+	57	14	13	4	59	51	198
Total	1157	109	103	56	283	185	1893
B. Row Percentages							
12	74.7	4.3	4.5	2.3	9.2	4.9	100.0
13	58.6	7.2	6.3	6.3	16.2	5.4	100.0
14	42.1	9.5	14.7	7.4	14.7	11.6	100.0
15	47.9	14.6	8.3	4.2	12.5	12.5	100.0
16	34.7	7.6	4.6	3.4	29.4	20.2	100.0
17+	28.8	7.1	6.6	2.0	29.8	25.8	100.0
Total	61.1	5.8	5.4	3.0	14.9	9.8	100.0
C. Column Percentages							
12	76.1	46.8	51.4	48.2	38.5	31.4	62.3
13	5.6	7.3	6.8	12.5	6.4	3.2	5.9
14	3.5	8.3	13.6	12.5	4.9	5.9	5.0
15	2.0	6.4	3.9	3.6	2.1	3.2	2.5
16	7.9	18.3	11.7	16.1	27.2	28.6	13.8
17+	4.9	12.8	12.6	7.1	20.8	27.6	10.5
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0
D. Estimated Frequencies Under Symmetry (G^2 = 11.59 with 15 df)							
12	881	58	46.5	25	100	57.5	1168
13	58	8	8	7	19	10	110
14	46.5	8	14	5.5	13	12	99
15	25	7	5.5	2	7.5	5	52
16	100	19	13	7.5	77	56	272.5
17+	57.5	10	12	5	56	51	191.5
Total	1168	110	99	52	272.5	191.5	1893
E. Row (or Column) Percentages Under Symmetry							
12	75.4	5.0	4.0	2.1	8.6	4.9	100.0
13	52.7	7.3	7.3	6.4	17.3	9.1	100.0
14	47.0	8.1	14.1	5.6	13.1	12.1	100.0
15	48.0	13.5	10.6	3.8	14.4	9.6	100.0
16	36.7	7.0	4.8	2.8	28.3	20.6	100.0
17+	30.0	5.2	6.3	2.6	29.2	26.6	100.0
Total	61.7	5.8	5.2	2.7	14.4	10.1	100.0

TABLE 20 (continued)

Respondent's Education	Sibling's Education 12	13	14	15	16	17+	Total
F. Observed/Expected Frequencies Under Independence (G^2 = 333.7 with 25 df)							
12	1.22	.75	.83	.77	.62	.50	
13	.96	1.25	1.16	2.13	1.08	.55	
14	.69	1.64	2.71	2.49	.99	1.18	
15	.78	2.53	1.53	1.41	.84	1.28	
16	.57	1.33	.84	1.16	1.97	2.07	
17+	.47	1.23	1.21	.68	1.99	2.64	
G. Multiplicative Parameters of the Saturated Row x Column Model							
12	2.48	.83	.99	.91	.82	.66	
13	1.33	.95	.95	1.71	.98	.50	
14	.73	.95	1.69	1.52	.66	.81	
15	.93	1.64	1.07	.96	.65	.98	
16	.72	.92	.63	.85	1.64	1.70	
17+	.62	.88	.93	.52	1.71	2.24	
H. Mosteller's Adjustment to Uniform Marginal Sums							
12	2.16	.79	.92	.82	.75	.57	6.00
13	1.19	.93	.92	1.59	.93	.44	6.00
14	.66	.93	1.63	1.42	.64	.72	6.00
15	.86	1.64	1.06	.92	.63	.89	6.00
16	.65	.90	.61	.80	1.54	1.51	6.00
17+	.52	.81	.85	.45	1.51	1.86	6.00
Total	6.00	6.00	6.00	6.00	6.00	6.00	36.00
I. Parameter Labels Under Multiplicative Model							
12	1	2	2	2	2	2	
13	2	2	2	2	2	2	
14	2	2	1	2	2	2	
15	2	2	2	2	2	2	
16	2	2	2	2	1	1	
17+	2	2	2	2	1	1	
J. Relative Densities Under Multiplicative Model (G^2 = 22.59 with 24 df)							
12	2.40	.75	.93	.77	.92	.75	
13	.97	.64	.67	1.09	.83	.42	
14	.80	.97	1.80	1.46	.86	1.04	
15	.79	1.30	.89	.72	.64	.98	
16	.87	1.02	.74	.89	2.26	2.38	
17+	.72	.95	1.05	.53	2.29	3.03	

Note: Parameter estimates under the model of Panel J are $\hat{\delta}_1$ = 2.40 and $\hat{\delta}_2$ = .84; $\hat{\delta}_1/\hat{\delta}_2$ = 2.85.

TABLE 21

Respondent's Broad Occupation Group by Friend's Occupation: Detroit Men (Laumann's Data from Jackson)

Friend's Occupation	Respondent's Occupation				
	1	2	3	4	Total
A. Frequencies					
1. Upper White Collar	329	162	84	87	662
2. Lower White Collar	226	284	123	165	798
3. Upper Blue Collar	83	103	265	218	669
4. Lower Blue Collar	55	97	174	418	744
Total	693	646	646	888	2873
B. Column Percentages					
1. UWC	47.5	25.1	13.0	9.8	23.0
2. LWC	32.6	44.0	19.0	18.6	27.8
3. UBC	12.0	15.9	41.0	24.5	23.3
4. LBC	7.9	15.0	26.9	47.1	25.9
Total	100.0	100.0	100.0	100.0	100.0
C. Observed/Expected Frequencies Under Independence (G^2 = 778.17 with 9 df)					
1. UWC	2.06	1.09	.56	.43	
2. LWC	1.17	1.58	.69	.67	
3. UBC	.51	.68	1.76	1.05	
4. LBC	.31	.58	1.04	1.82	
D. Estimated Frequencies Under Quasi-Symmetry (G^2 = 2.25 with 3 df)					
1. UWC	329.0	168.3	83.7	81.0	662
2. LWC	219.7	284.0	128.3	166.1	798
3. UBC	83.3	97.8	265.0	223.0	669
4. LBC	61.0	95.9	169.0	418.0	744
Total	693	646	646	888	2873
E. Starting Values Under Quasi-Perfect Choice (Blocked Diagonal)					
1. UWC	0	1	1	1	
2. LWC	1	0	1	1	
3. UBC	1	1	0	1	
4. LBC	1	1	1	0	
F. Observed/Expected Frequencies Under Quasi-Perfect Choice (G^2 = 188.92 with 2 df)					
1. UWC	3.57*	1.55	.81	.67	
2. LWC	1.47	1.61*	.77	.84	
3. UBC	.72	.72	2.05*	1.47	
4. LBC	.62	.82	1.53	3.22*	

*Cell ignored in estimating the model.

TABLE 21 (continued)

Friend's Occupation	Respondent's Occupation 1	2	3	4	Total
G. Parameter Labels Under Multiplicative Model					
1. UWC	5	2	1	1	
2. LWC	2	2	1	1	
3. UBC	1	1	3	2	
4. LBC	1	1	2	4	
H. Relative Densities Under Multiplicative Model (G^2 = 6.46 with 5 df)					
1. UWC	3.27	1.22	.67	.60	
2. LWC	1.29	1.23	.57	.66	
3. UBC	.66	.62	1.71	1.21	
4. LBC	.50	.67	1.29	2.66	
I. Starting Values Under Model of Linear Density Gradient (G^2 = 7.63 with 9 df)					
1. UWC	5	2	1	1	
2. LWC	2	2	1	1	
3. UBC	1	1	3	2	
4. LBC	1	1	2	4	

Note: Parameter estimates under the model of Panel H are $\hat{\delta}_1 = .622$, $\hat{\delta}_2 = 1.245$, $\delta_3 = 1.712$, $\delta_4 = 2.664$, $\delta_5 = 3.268$.

TABLE 2?

Tests for Change in Mobility from Father's (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation, U.S. Men Aged 20-64 in 1973 by 5-Year Age Cohorts

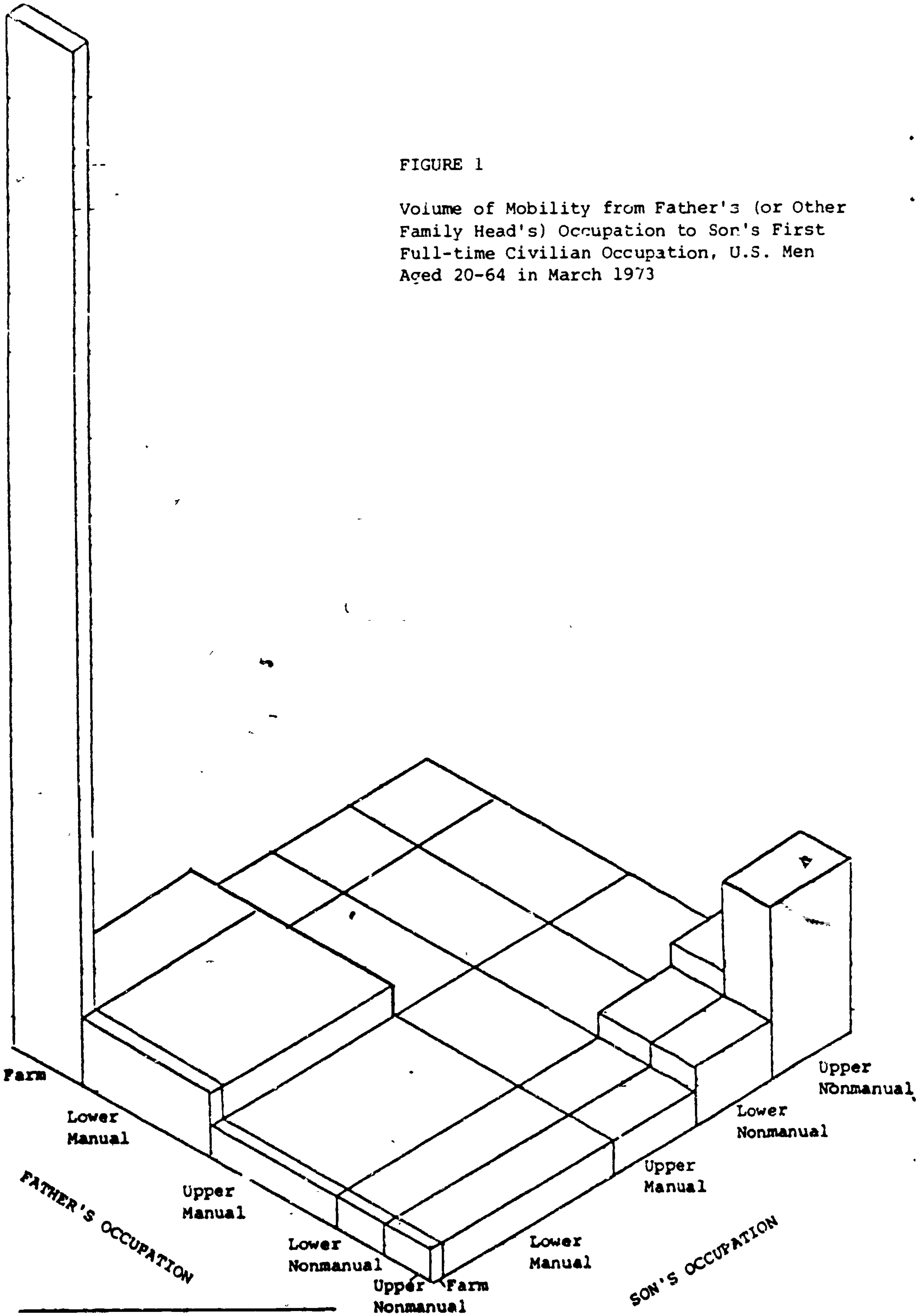
	Model				Change				
	(PA) (WA) (H)		(PA) (WA) (HA)		Unblocked levels		Blocked level		
Level blanked out	G^2	df	G^2	df	G^2	df	G^2	df	α
None	235.3	140	175.6	108	59.7	32	--	--	--
1	229.2	132	175.6	108	53.6	24	6.10	8	>.500
2	216.6	132	175.6	108	41.0	24	18.7	8	<.025
3	233.3	132	175.6	108	57.7	24	2.0	8	>.500
4	165.0	96	140.8	72	24.2	24	35.5*	8	<.001
5	11.9	24	0.0	0	11.9	24	47.8*	8	<.001

NOTES: P = father's occupation, W = son's first occupation, A = Age, H = design matrix from Table 4.

*Significant at the α = .05 level in a simultaneous test; critical χ^2 = 20.09, given the 5 tests reported here.

FIGURE 1

Volume of Mobility from Father's (or Other Family Head's) Occupation to Son's First Full-time Civilian Occupation, U.S. Men Aged 20-64 in March 1973



NOTE: See text for explanation.

FIGURE 2

Schematic Arrangement of Likelihood-Ratio Test Statistics (G^2) for Change in Interaction Parameters and Lack of Fit of Multiplicative Models

Entires at kth level	Interaction parameters: Constant	Interaction parameters: Variable
Fitted under the model	G_a^2: change and lack of fit at all levels of the design	G_b^2: lack of fit at all levels of the design
Blanked out	G_c^2: change and lack of fit at all but the k^{th} level of the design	G_d^2: lack of fit at all but the k^{th} level of the design

FIGURE 3

Test Statistics for Selected Hypotheses About Interaction in Two or More Classifications

Null hypothesis	Test statistic	Degrees of freedom
1. No change in interaction parameters at any level of the design	$G_a^2 - G_b^2$	$(L-1)(K-1)$
2. No change in interaction parameters at any but the k^{th} level of the design	$G_c^2 - G_d^2$	$(L-1)(K-2)$
3. No change or lack of fit at the k^{th} level of the design	$G_a^2 - G_c^2$	LM_k-1
4. No lack of fit at the k^{th} level of the design	$G_b^2 - G_d^2$	$L(M_k-1)$
5. No change in interaction parameters at the k^{th} level of the design	$(G_a^2 - G_b^2) - (G_c^2 - G_d^2)$	$L-1$

NOTE: See Figure 2 for definitions of G_a^2, G_b^2, G_c^2, G_d^2.

L = number of cross-classifications compared,
K = number of levels of the design matrix,
M_k = number of independently variable cells at the k^{th} level of the design matrix.